En-Var in NWP (and ocean models)

Nils Gustafsson, SMHI

Many thanks to Jelena Bojarova and Lars Axell!

Basic problems of data assimilation

Filtering,

Interpolation

and

Balancing

Estimation of the characteristics of forecast errors (predictability) is important for all these three aspects of data assimilation !

HIRLAM first approach to use ensembles in 3D-Var and 4D-Var

- Use the ETKF algorithm for re-scaling of a 6h forecast ensemble to an analysis ensemble (estimation of the analysis error covariance).
- Use ensemble of 3h (4D-Var) or 6h (3D-Var) forecasts to estimate the background error covariance and blend it with the static background error covariance => Hybrid variational ensemble data assimilation, a step towards 4D-En-Var

Why 4D-En-Var?

- **Avoid use of TL and AD models (low resolution TL and AD models have difficulties to scale on thousands of processors) => 4D-En-Var is cheaper than 4D-Var**
- **Utilizes 4D ensemble perturbations based on the non-linear model.**
- **Easy to implement with existing 4D-Var Hybrid**
- **4D-En-Var in its simplest form is similar to 4D-En-KF. 4D-En-Var has possibilities to treat non-linearities better (outer loops); Easy to add 3D-Var FGAT background error constraint.**

Incremental 4D-Var

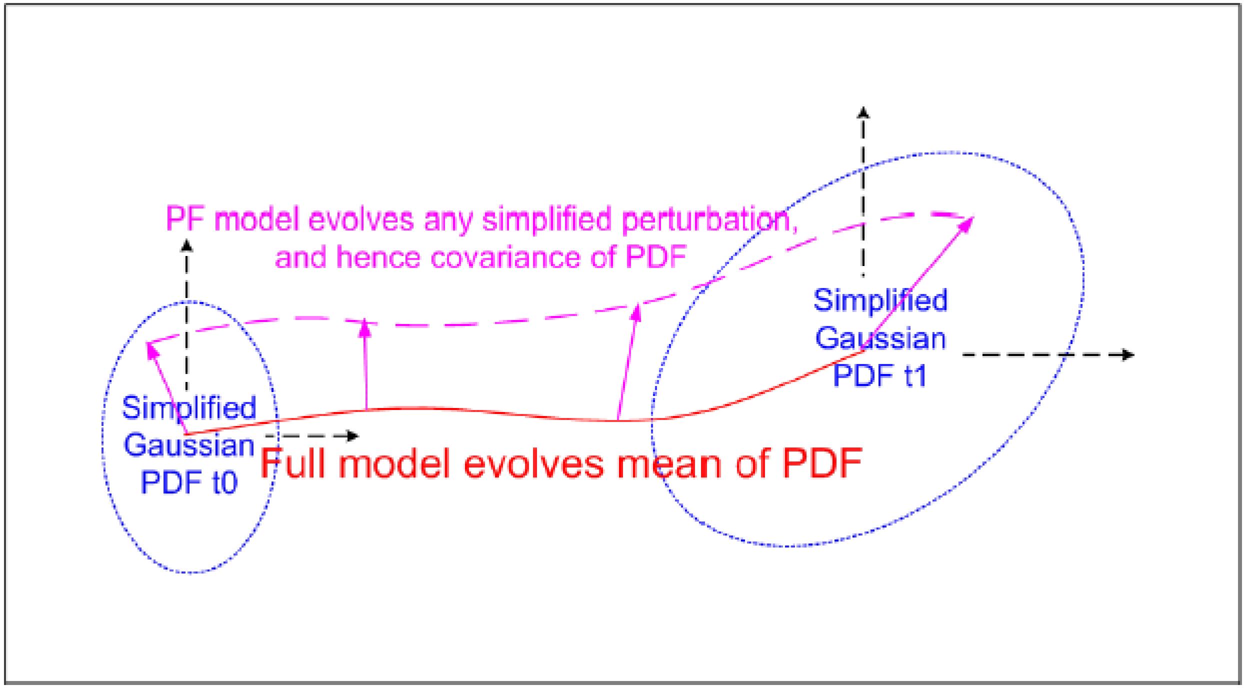


Figure 3: Statistical, incremental, 4D-Var approximates entire PDF by a Gaussian. The 4D analysis increment is a trajectory of the PF model, optionally augmented by a model error correction term.

From Lorenc (2011)

4D-En-Var

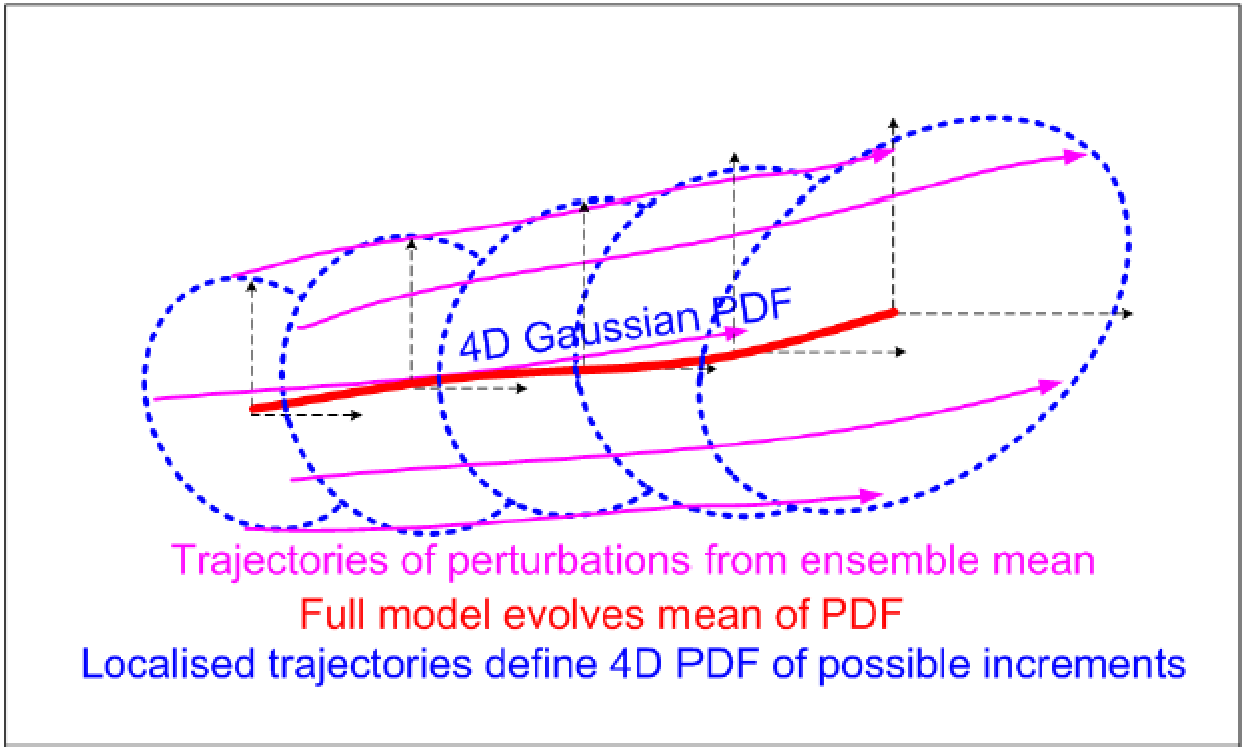


Figure 6: A schematic diagram of 4D-En-Var, for comparison with figure 3. The 4D analysis is a localised linear combination of model trajectories – it is not itself a model trajectory.

From Lorenc (2011)

Incremental 4D-Var

Cost function minimized with respect to the increment $\delta\mathbf{X}$:

$$J = J_b + J_o = \frac{1}{2}(\delta\mathbf{X})^T\mathbf{B}^{-1}\delta\mathbf{X} + \frac{1}{2}\sum_{t_k=t_0}^{t_K}(\mathbf{H_k M_k}\delta\mathbf{X} - \mathbf{d}_k)^T\mathbf{R}_k^{-1}(\mathbf{H_k M_k}\delta\mathbf{X} - \mathbf{d}_k)$$

$\mathbf{B}$ the background error covariance
$t_k = t_0, ..., t_K$ the data assimilation time window
$\mathbf{d}_k = \mathbf{y}_k - H_k(M_k(\mathbf{x}_b))$ the innovations; $\mathbf{y}_k$ the vector of observations at time t_k
$\mathbf{x}_b$ the model background state valid at time t_0
$M_k(.)$ the non-linear model; $\mathbf{M_k}$ the corresponding tangent linear model
$H_k(.)$ is the non-linear observation operator; $\mathbf{H_k}$ the linearized observation operator
$\mathbf{R}_k$ is the observation error covariance

Introduce a pre-conditioning matrix $\mathbf{U}$ such that $\mathbf{B} = \mathbf{U}\mathbf{U}^T$, $\delta\mathbf{X} = \mathbf{U}\chi$

The cost function to be minimized and its gradient with respect to the assimilation control variable χ are given by:

$$J = J_b + J_o = \frac{1}{2}\chi^T\chi + \frac{1}{2}\sum_{t_k=t_0}^{t_K}(\mathbf{H_k M_k U}\chi - \mathbf{d}_k)^T\mathbf{R}_k^{-1}(\mathbf{H_k M_k U}\chi - \mathbf{d}_k)$$

and

$$\nabla_\chi J = \chi + \sum_{t_k=t_0}^{t_K}\mathbf{U}^T\mathbf{M_k^T H_k^T}\mathbf{R}_k^{-1}(\mathbf{H_k M_k U}\chi - \mathbf{d}_k)$$

4D-Ens-Var (no localization)

Replace the static error covariance $\mathbf{B}$ with a flow-dependent error covariance $\mathbf{B} \approx \mathbf{X}'_\mathbf{b}(\mathbf{X}'_\mathbf{b})^T$ estimated from an ensemble of background model states

$\mathbf{X}'_\mathbf{b}$ is a matrix whose columns are the normalized deviations of the ensemble background states from their mean:

$$\mathbf{X}'_\mathbf{b} = \frac{1}{\sqrt{N-1}}(\mathbf{X}_\mathbf{b1} - \overline{\mathbf{X}_\mathbf{b}}, \ldots.., \mathbf{X}_\mathbf{bN} - \overline{\mathbf{X}_\mathbf{b}})$$

N is the number of ensemble members.

Apply $\mathbf{X}'_\mathbf{b}$ for the pre-conditioning $\delta\mathbf{X} = \mathbf{X}'_\mathbf{b}\chi$

Use an ensemble of non-linear model integrations over the data assimilation window and apply the following approximation in the observation constraint part of the cost function:

$$\mathbf{H}_\mathbf{k}\mathbf{M}_\mathbf{k}\mathbf{X}'_\mathbf{b} \approx \frac{1}{\sqrt{N-1}}(\mathbf{H}_\mathbf{k}(M_k(\mathbf{X}_\mathbf{b1})) - M_k(\overline{\mathbf{X}_\mathbf{b}})), \ldots\ldots., \mathbf{H}_\mathbf{k}(M_k(\mathbf{X}_\mathbf{bN})) - M_k(\overline{\mathbf{X}_\mathbf{b}})))$$

The forward integration of the tangent linear model is replaced by the non-linear model trajectories.

The gradient of the cost function now becomes:

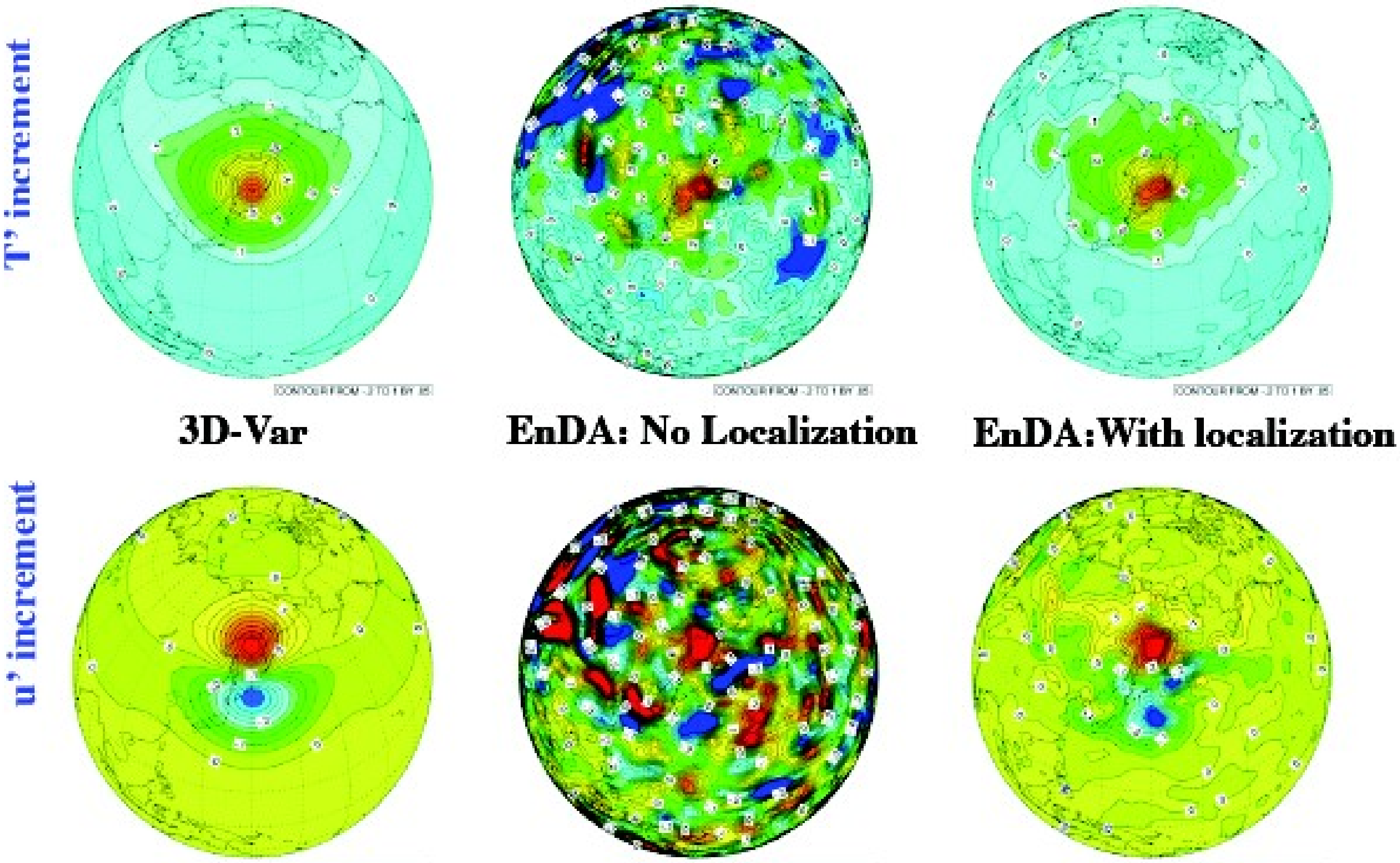
$$\nabla_\chi J = \chi + \sum_{t_k=t_0}^{t_K} (\mathbf{H}_\mathbf{k}\mathbf{M}_\mathbf{k}\mathbf{X}'_\mathbf{b})^T \mathbf{R}_k^{-1}(\mathbf{H}_\mathbf{k}\mathbf{M}_\mathbf{k}\mathbf{X}'_\mathbf{b}\chi - \mathbf{d}_k)$$

From Dale Barker (2011)

Alpha Covariance Localization (Extreme example: 1 ob + 2 members!)

Met Office

- Single T observation (O-B, s_o=1K) at 50N, 150E, 500hPa.



4D-En-Var (with localization)

En-KF: Covariance localization by an element-by-element multiplication (Schur product) with a full rank localization correlation matrix $\mathbf{C}$:

$\mathbf{B} \approx \mathbf{C} \circ \mathbf{B}_{ens} = \mathbf{C} \circ \mathbf{X}'_{\mathbf{b}}(\mathbf{X}'_{\mathbf{b}})^T$

$\mathbf{C}$ is constructed such that the final covariances $\mathbf{B} = \mathbf{C} \circ \mathbf{B}_{ens}$ will be zero over distances longer than a pre-defined localization length scale.

For 4D-Ens-Var the pre-conditioning can be done with the matrix $\mathbf{P}'_{\mathbf{b}}$ given by

$$\mathbf{P}'_{\mathbf{b}} = (\mathbf{C}' \circ \mathbf{X}'_{\mathbf{b1}}, \mathbf{C}' \circ \mathbf{X}'_{\mathbf{b2}}, ..., \mathbf{C}' \circ \mathbf{X}'_{\mathbf{bN}})$$

$\mathbf{C}'\mathbf{C}'^T = C$
$\mathbf{X}'_{\mathbf{bk}}$ is an n-column matrix with every column being equal to the k^{th} column in $\mathbf{X}'_{\mathbf{b}}$
N is the number of ensemble members
n is the model space dimension.
$\mathbf{C}'$ consist of eigenvectors of the correlation matrix $\mathbf{C}$.

$$J = \frac{1}{\beta_{3dvar}} J_{3dvar} + \frac{1}{\beta_{ens}} J_{ens} + J_o$$

4D-Var:

$$\delta x(t_k) = \mathbf{M_k} \delta x(t_0)$$

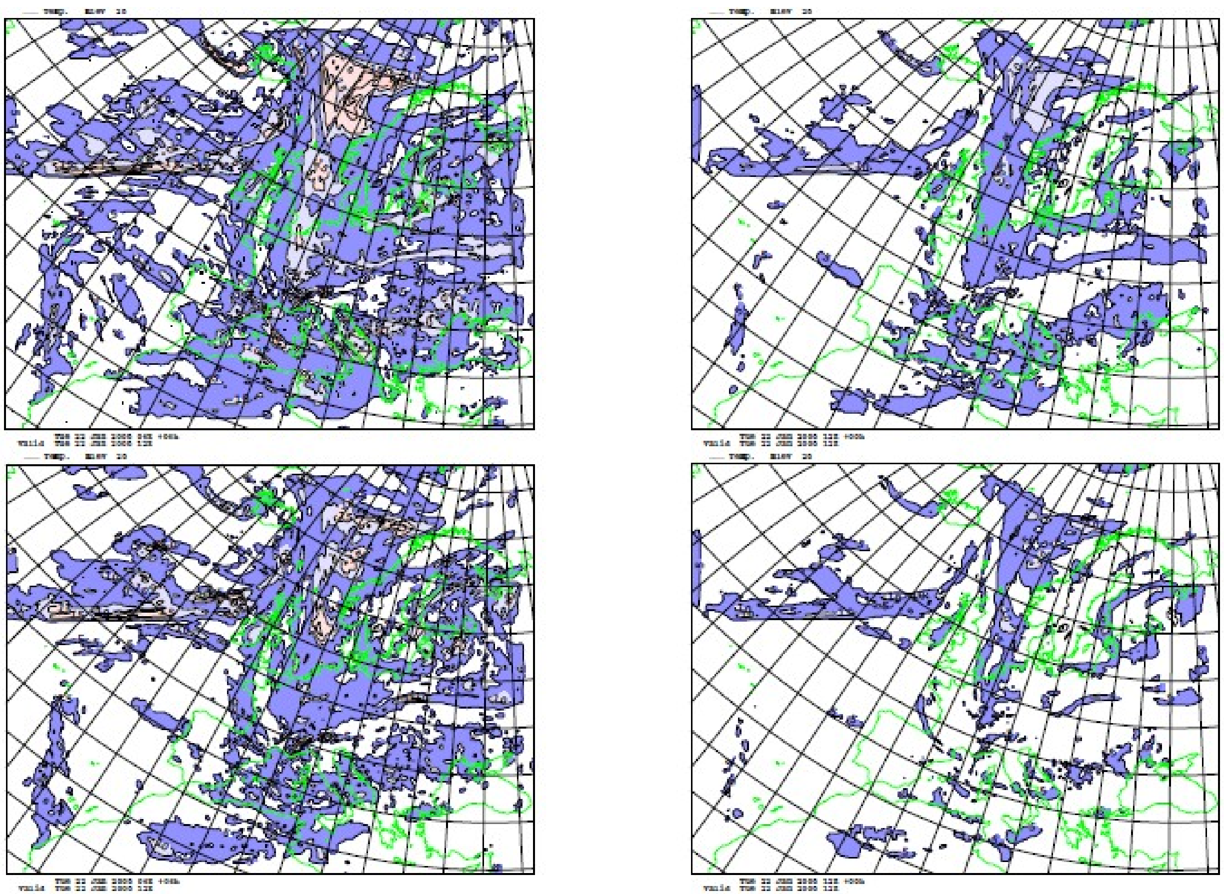
4D-Var Hybrid:

$$\delta x(t_k) = \mathbf{M_k}(\delta x^{3dvar}(t_0) + \sum_{l=1}^{N} \alpha_l \delta x_l^{ens}(t_0))$$

4D-En–Var:

$$\delta x(t_k) = \delta x^{3dvar}(t_0) + \sum_{l=1}^{N} \alpha_l \delta x_l^{ens}(t_k))$$

Examples of ensemble spread (standard deviation) for temperature at model level 28 (~800 hPa)



3D-Var

4D-Var

Figure 12. Temperature level 28 spread (rms), 3dvar (top), 4dvar(bottom), before etkf re-scaling (left), after etkf re-scaling (right), 22

Before ETKF re-scaling

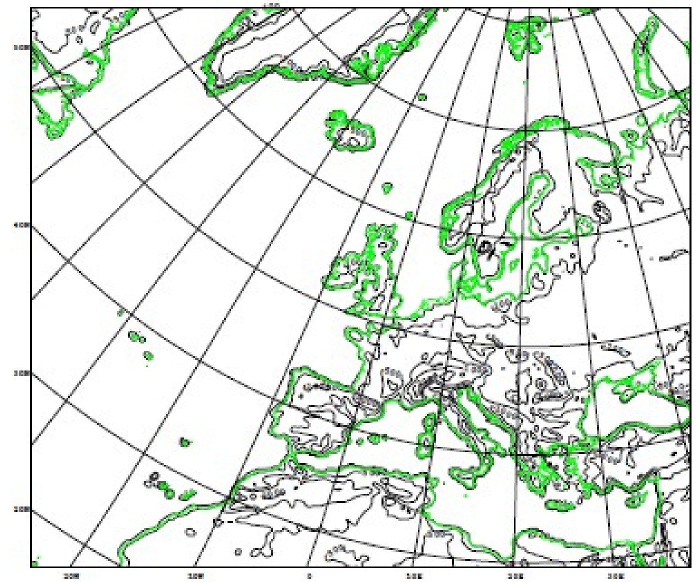
After ETKF re-scaling

Experiments over 17 January – 29 February 2008

4dvar_ref1: 4D-Var, 2 outer loops (6 h window, 20 iter. at 66 km and 40 iter. at 44 km incr. resol.), simple TL physics (vertical diffusion only), J_c DFI

4dvar_hybrid1: As 4dvar_ref1 with hybrid ensemble constraint, 20 members, ETKF perturb., 75% static and 25% ensemble variance, ens. perturbations inflated by a factor 4 in hybrid.

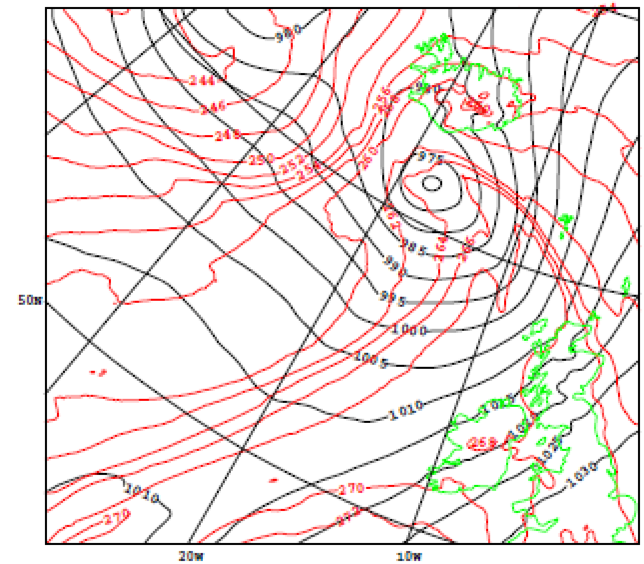
4DEnVar: 6 h window, 1 outer loop (60 iter. at 33 km incr. resol.). 50% static and 50% ens. variance, no ens. perturb. inflation, 3D-Var constraint in the middle of the window (<=> FGAT).



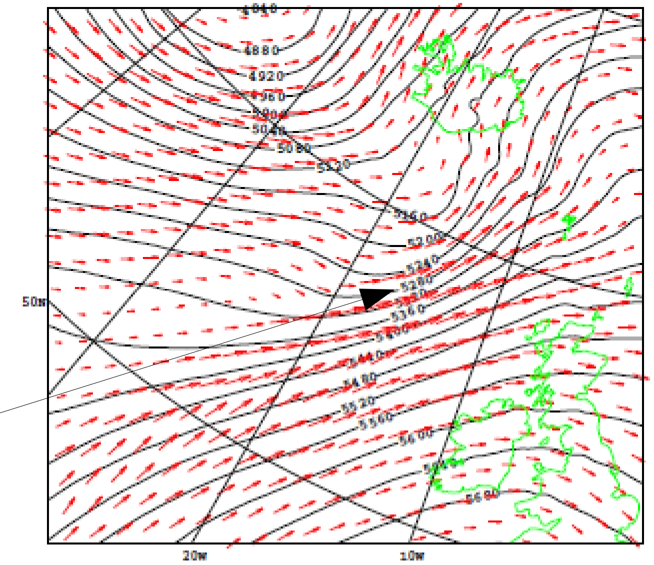
Model grid res. 11 km
40 levels
20 members

Example of single observation experiments with 4D-Var, 4D-Var Hybrid and 4D-En-Var

Background states



PMSL
T700



Z500
V500

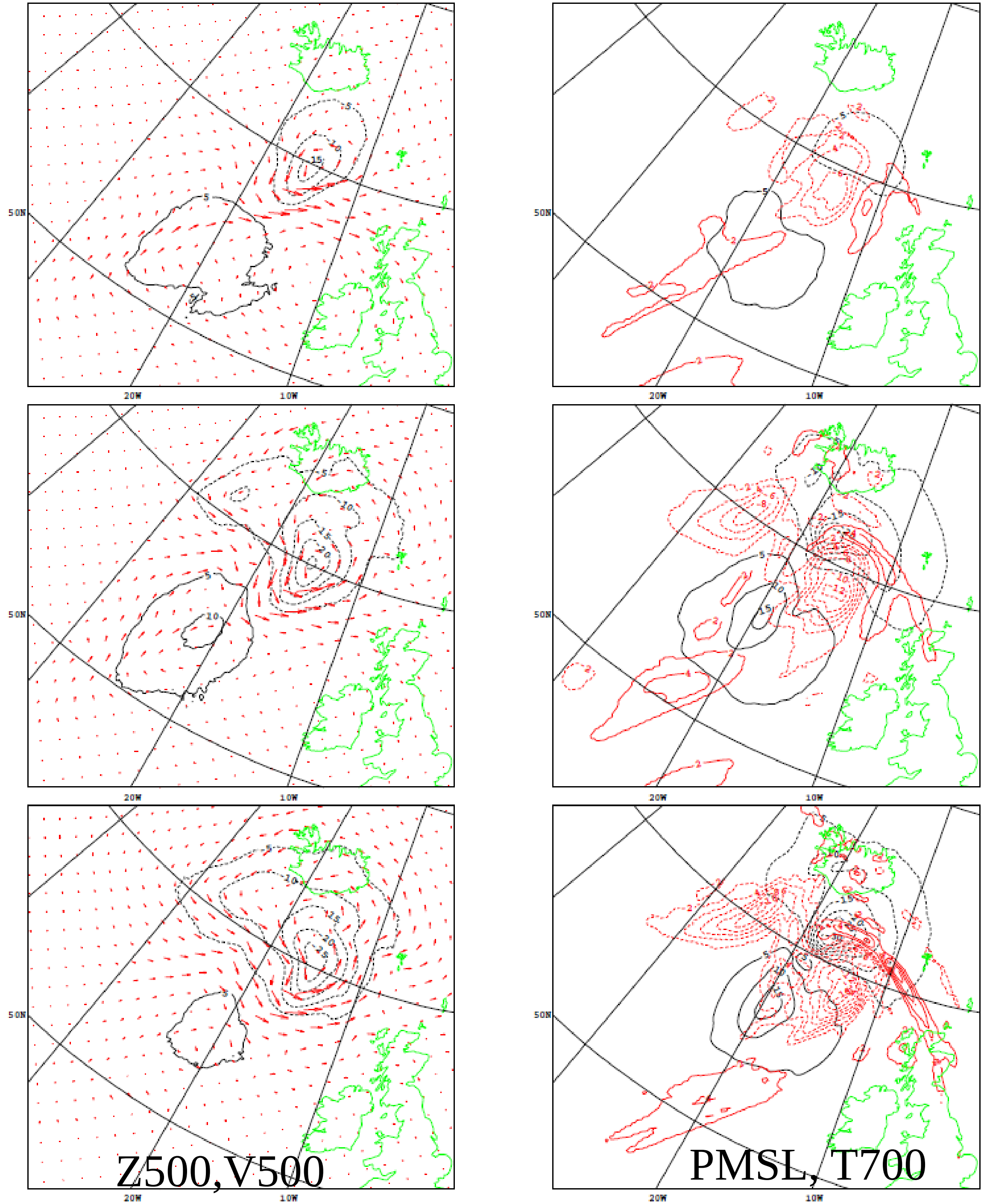
Position of simulated observation V500

Single observation assimilation increments

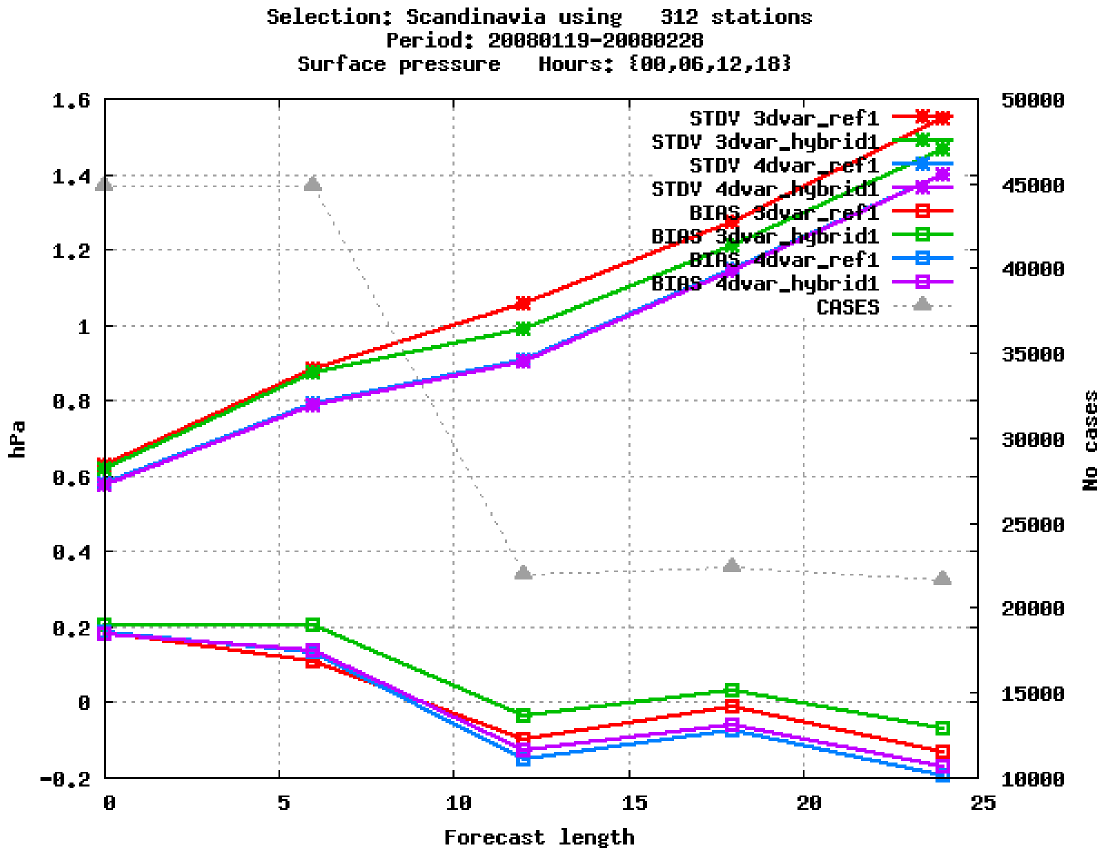
4D-Var

4D-Var Hybrid

4D-En-Var



Hybrid impact on forecast verification scores – mean sea level pressure

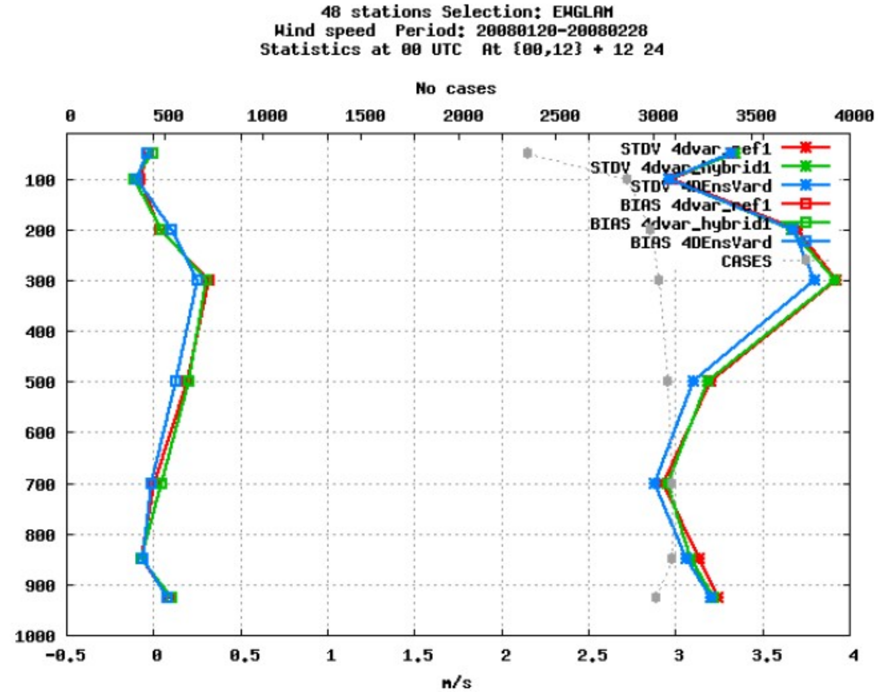


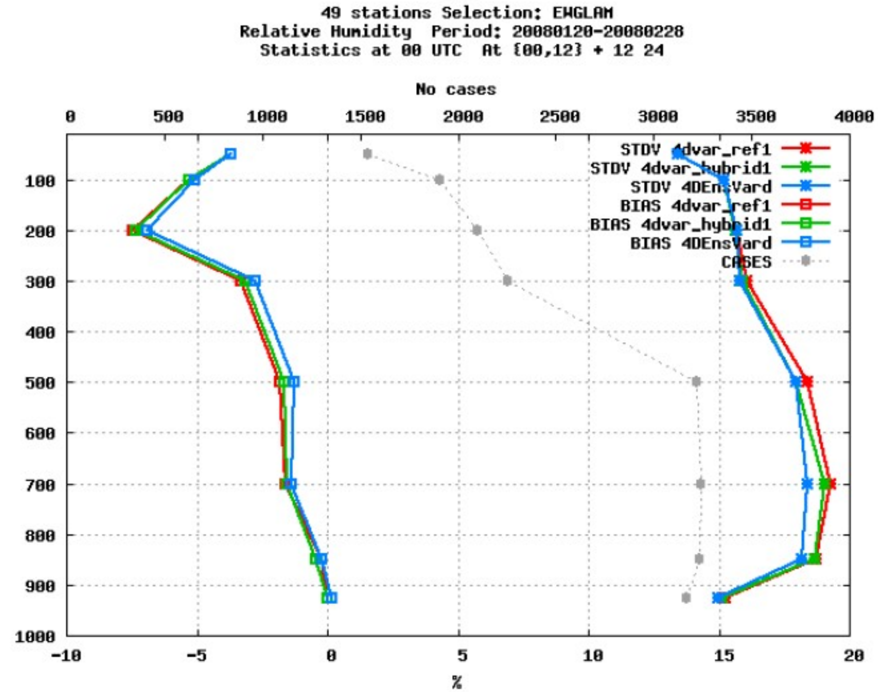
---- 3D-Var; ---- 3D-Var hybrid

---- 4D-Var; ---- 4D-Var hybrid

Verification of wind speed and relative humidity from 6 weeks of parallel runs with HIRLAM 4D-Var, 4D-Var Hybrid and 4D-En-Var; 20 ensemble members.

Note that 4D-En-Var is much cheaper since TL and AD models are not needed (provided an ensemble exists!).

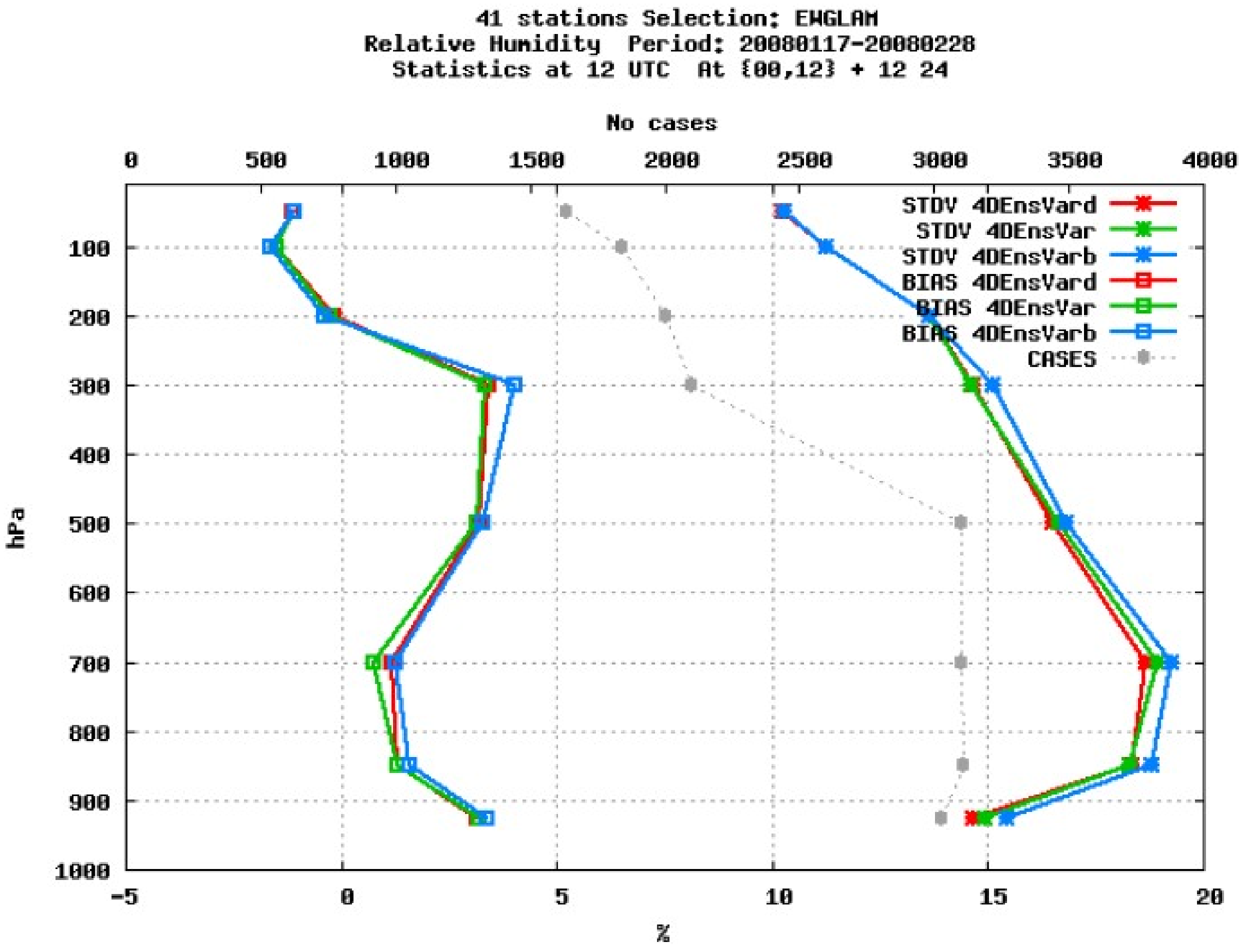




Sensitivity experiments

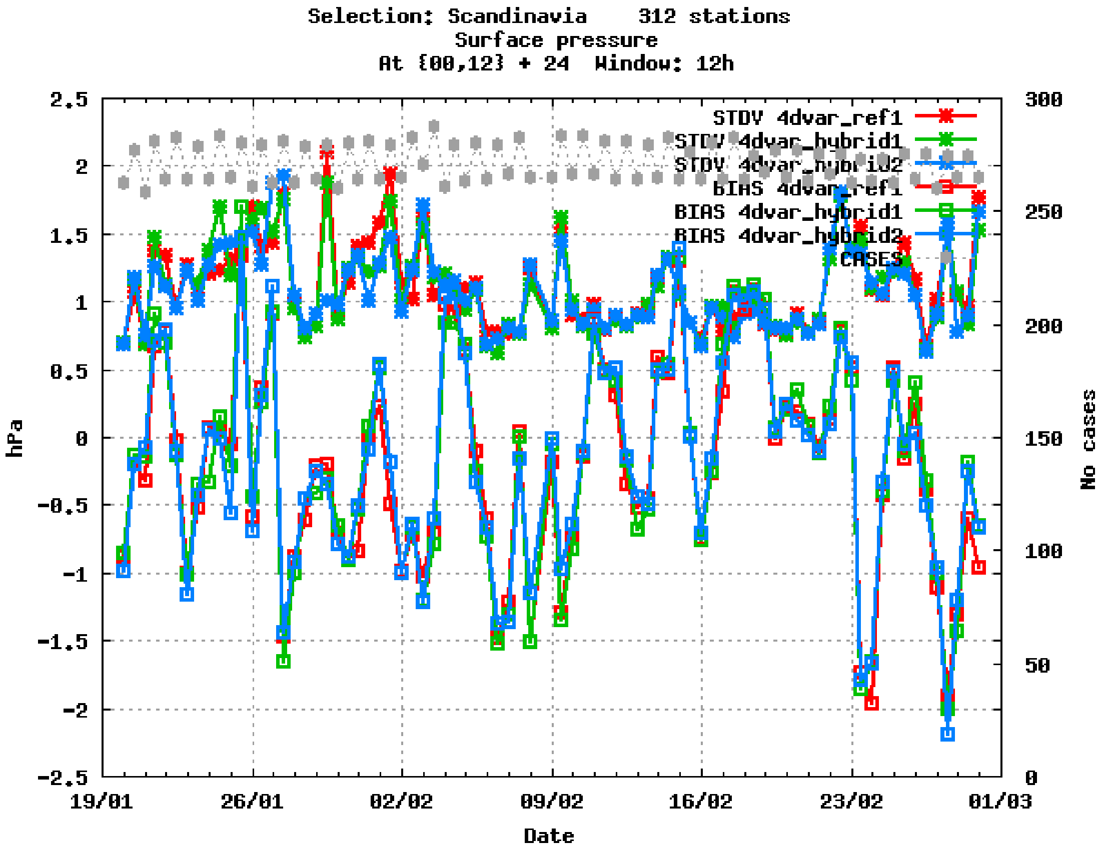
Is (further) inflation of ensemble perturbations needed?
4densvard (inflation 4.0) versus 4densvar (no inflation).
Answer: No!

Is static back-ground error constraint needed ?
4densvar (50%static) versus 4densvarb (10% static)
Answer: Yes (with 20 members only) !



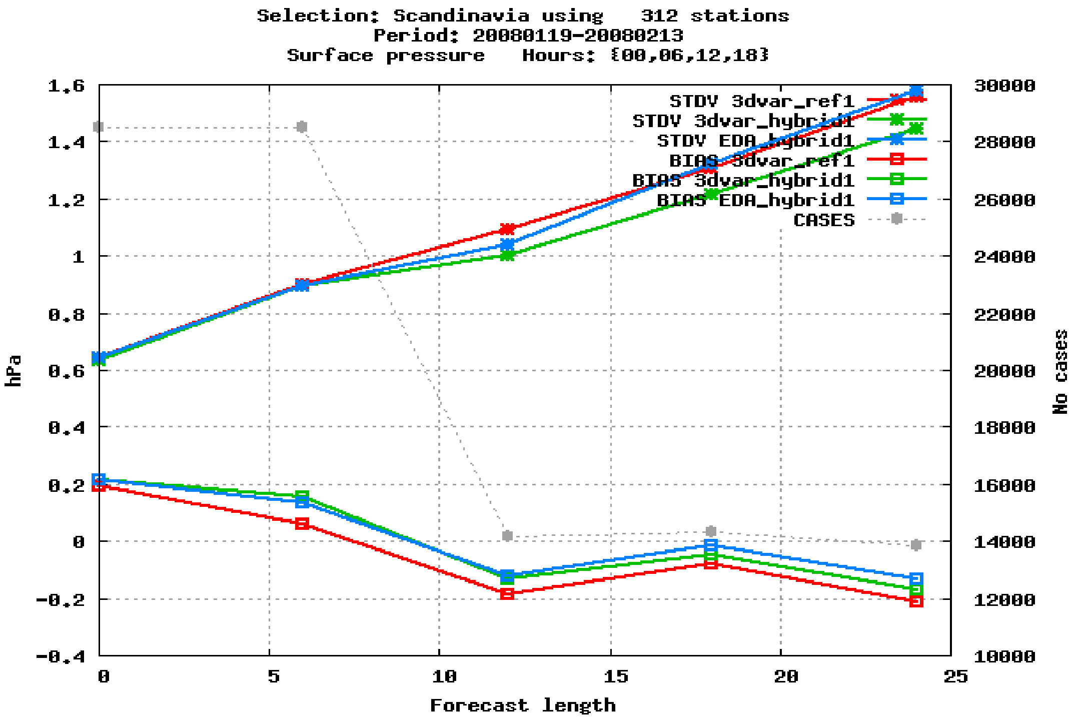
"Correcting phase errors"

4dvar_hyb1: the 4DVAR hybrid (ETKF with 20 members)
4dvar_hyb2: the 4DVAR hybrid (ETKF with 40 members:
20 members: fc20080122_06+003
20 members: fc20080122_06+005)



Which ensemble generation technique is better?

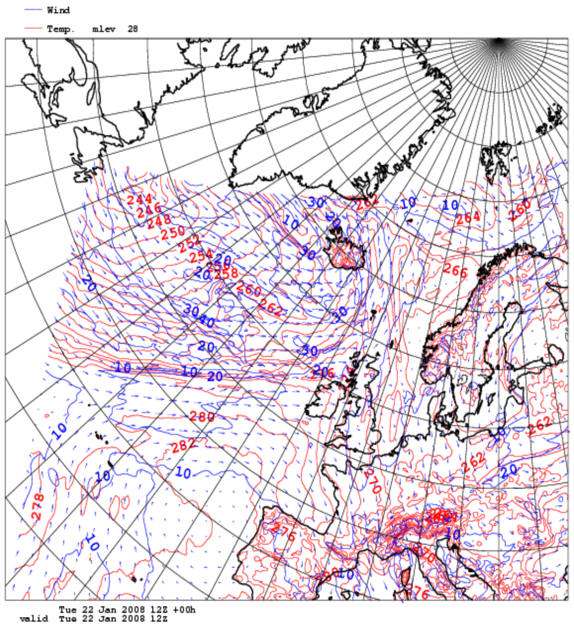
ETKF or EDA



3DVAR-ETKF outperforms ***3DVAR*** and is slightly better than ***3DVAR_EDA***

Dynamically consistent structures are important

EDA: analysis at 22 Jan 2008 12 UTC & mbr005



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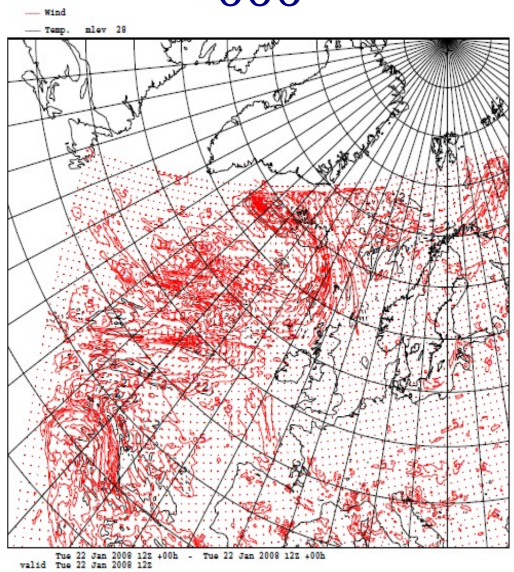

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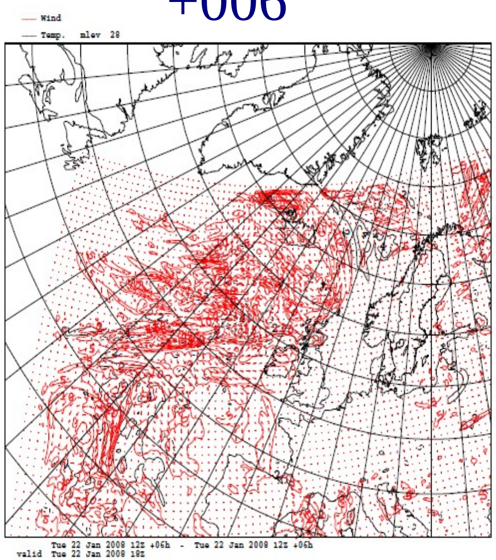

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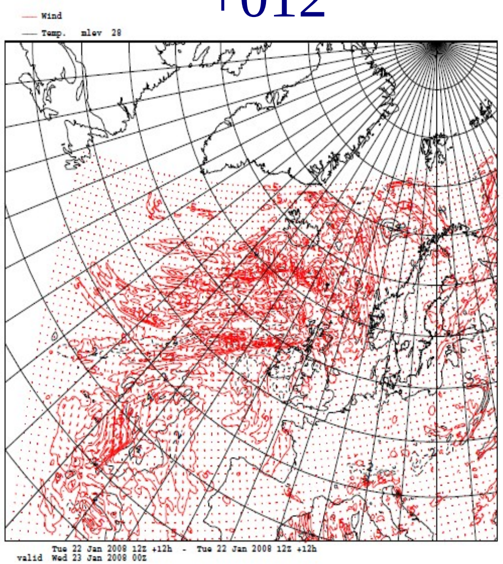

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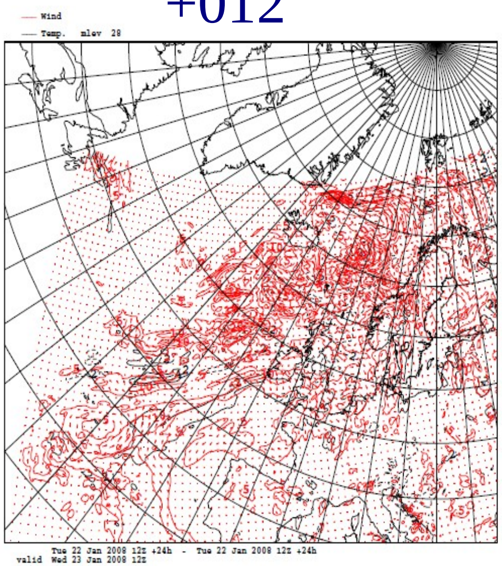
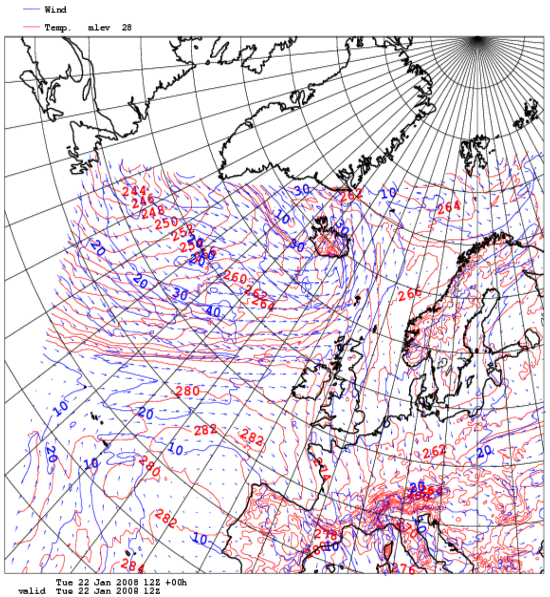


ETKF: analysis at 22 Jan 2008 12 UTC & mbr005



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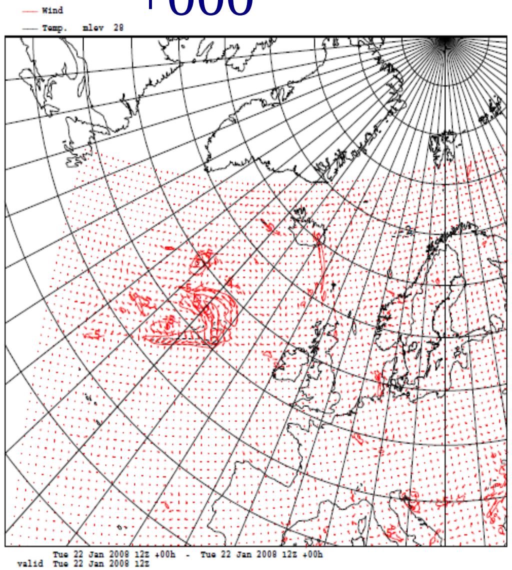

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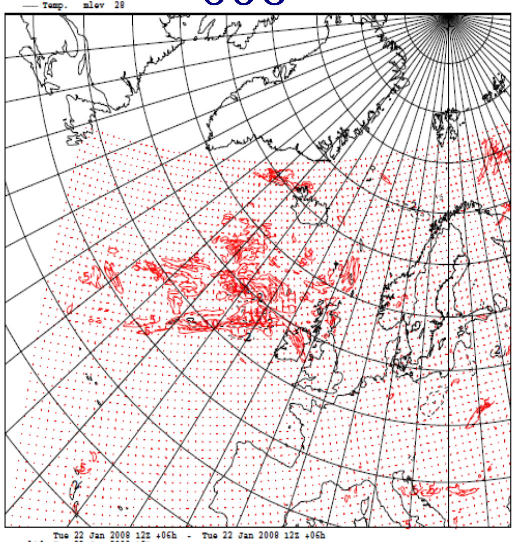

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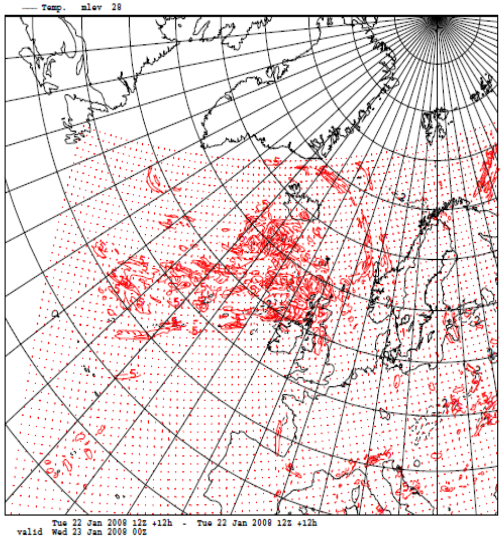

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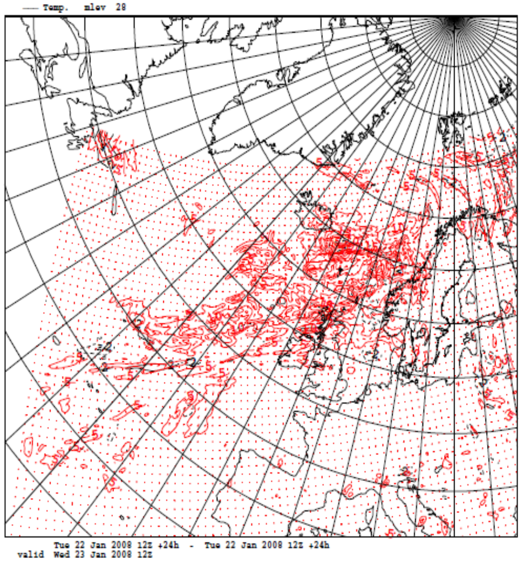


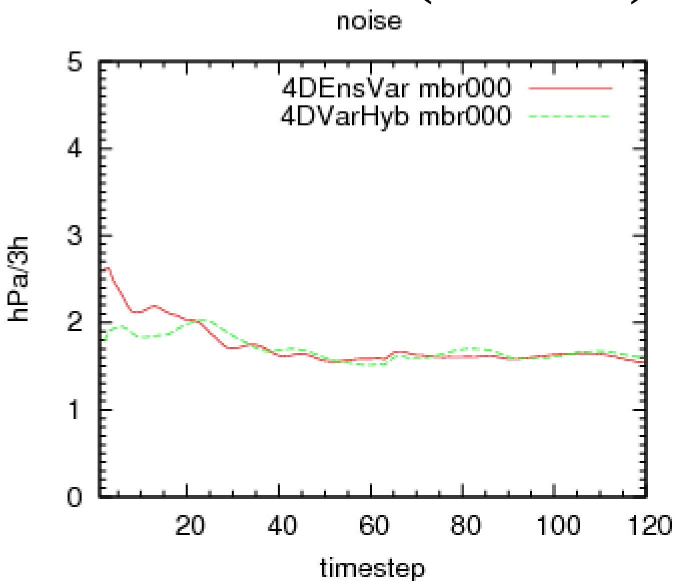
Is noise a potential problem for 4DEnVar (and ETKF re-scaling)?

- **A weak digital filter constraint is applied in HIRLAM 4D-Var and HIRLAM 4D-Var Hybrid for the control forecast – no explicit initialization is applied.**
- **Do we need to apply initialization (incremental DFI) after ETKF re-scaling for ensemble members other than the control ?**
- **Do we need to apply initialization after 4D-En-Var, which is a hybrid of 3D-Var FGAT increment and localized ETKF non-linear model perturbations ?**

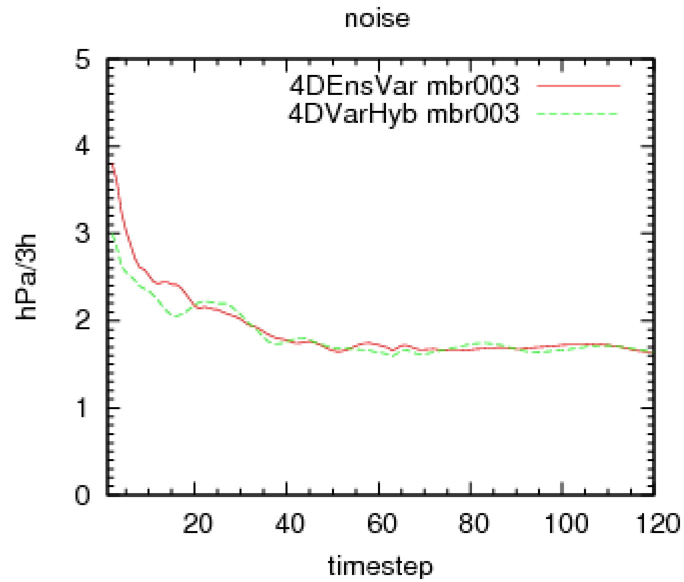
Average absolute surface pressure tendecies (hPa/3h) for forecasts starting from the main observation hour 22 February 2008 12UTC:

-------- 4D-Var Hybrid ------- 4DEnsVar

Member 0 (Control)



Member 3



- 4D-Var Hybrid Control is essentially noise-free
- 4dEnsVar control has a slightly incresed noise level
- Noise based on 4DEnsVar control increments and ETKF re-scaling of ensemble perturbations adds up

Publications:

Bojarova, J., Gustafsson, N., Johansson, Å. and Vignes, O., 2010:
The ETKF rescaling scheme in HIRLAM.
Tellus, **63A**, 385-401.

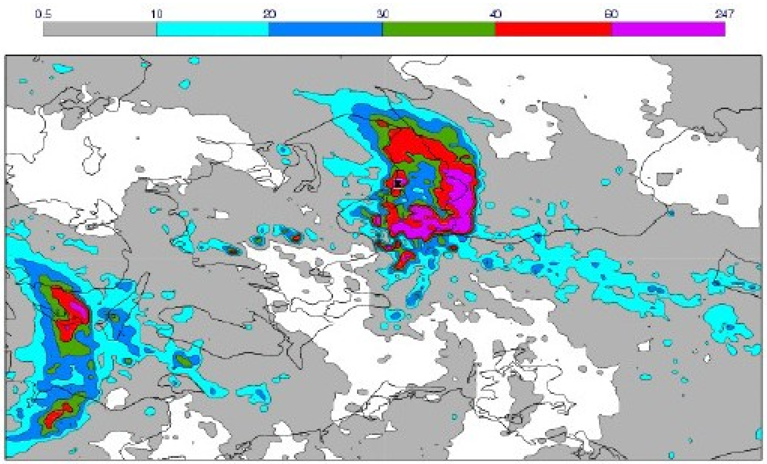
Gustafsson, N., Bojarova, J. and Vignes, O., 2014:
A hybrid variational ensemble data assimilation for the HIgh Resolution Limited Area Model (HIRLAM).
J. of Nonlin. Processes in Geophys..

Gustafsson, N. and Bojarova, J., 2014:
4-Dimensional Ensemble Variational (4D-En-Var) data assimilation for the HIgh Resolution Limited Area Model (HIRLAM).
J. of Nonlin. Processes in Geophys..

Verification of the HARMONIE AROME 2.5km forecasts for extreme weather event

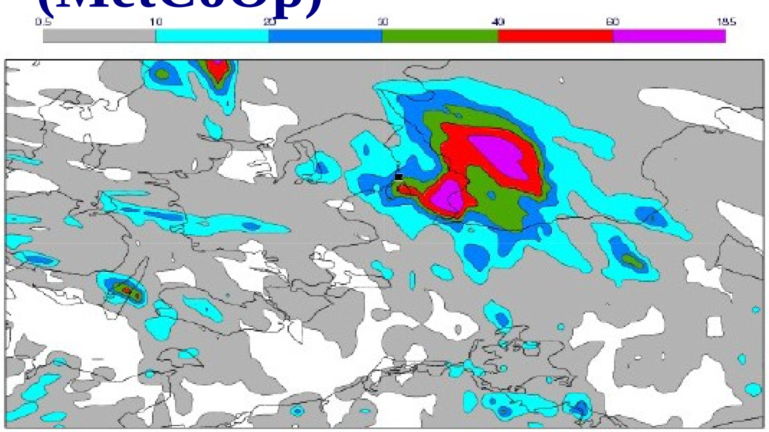
(from Xiaohua Yang (DMI) & Lisa Bengtsson et al (SMHI))

Radar data 31.08 00UTC - 12UTC



12 h accumulated rainfall

HARMONIE AROME + 30h (MetCoOp)



The HARMONIE AROME **is capable** in many cases to predict convective precipitation events (severe high impact weather events);

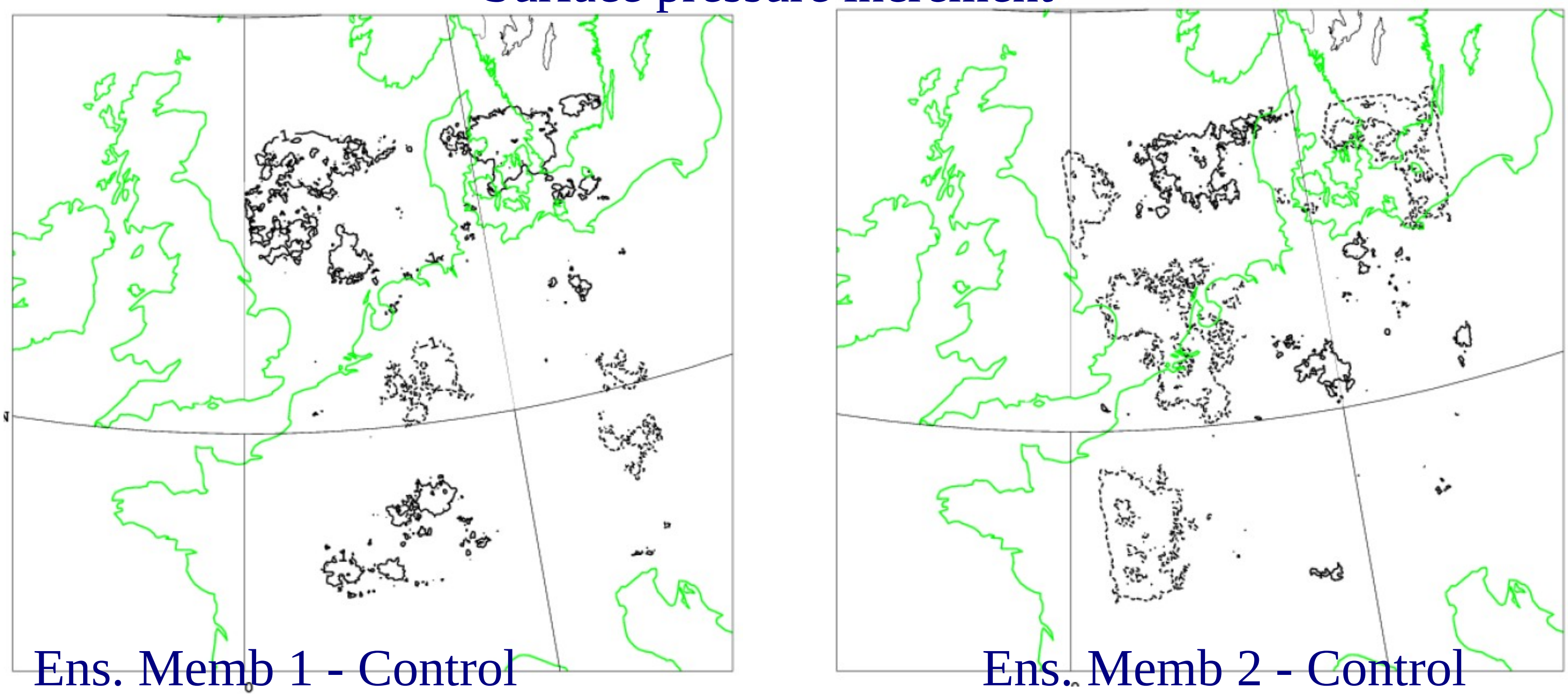
Stochastic nature of the convective phenomena should be taken into account both for verification and in post-processing (timing and location uncertainty);

The quality of the short-term forecasts in the operational runs is not satisfactory : **coupling strategy and data assimilation to be blamed**

Evolution of two random perturbations with structure of B-matrix covariance

Forecast length: +00h

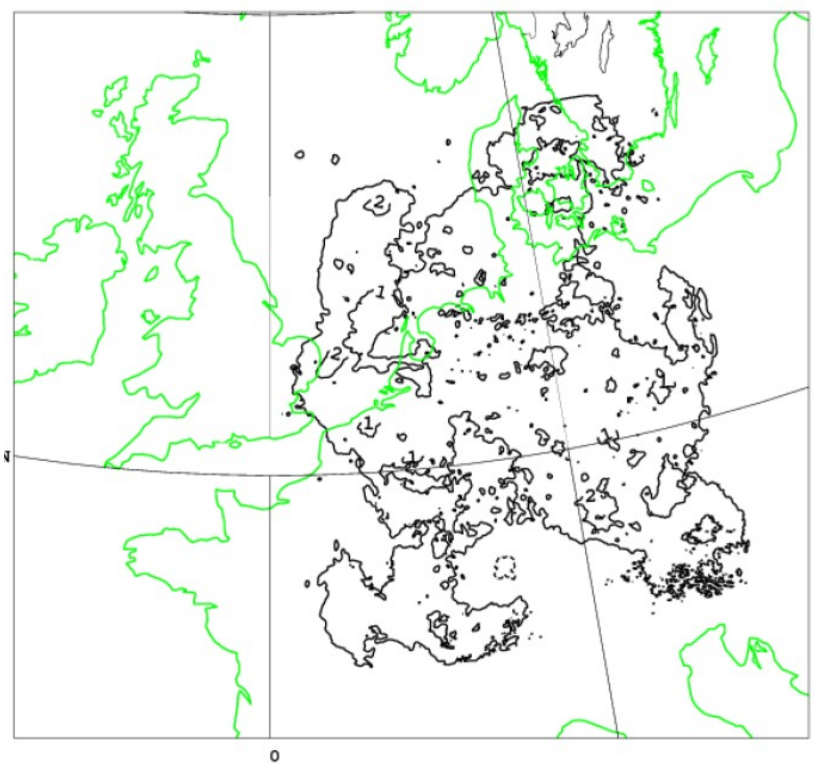
Surface pressure increment **13 08 2012 03UTC**

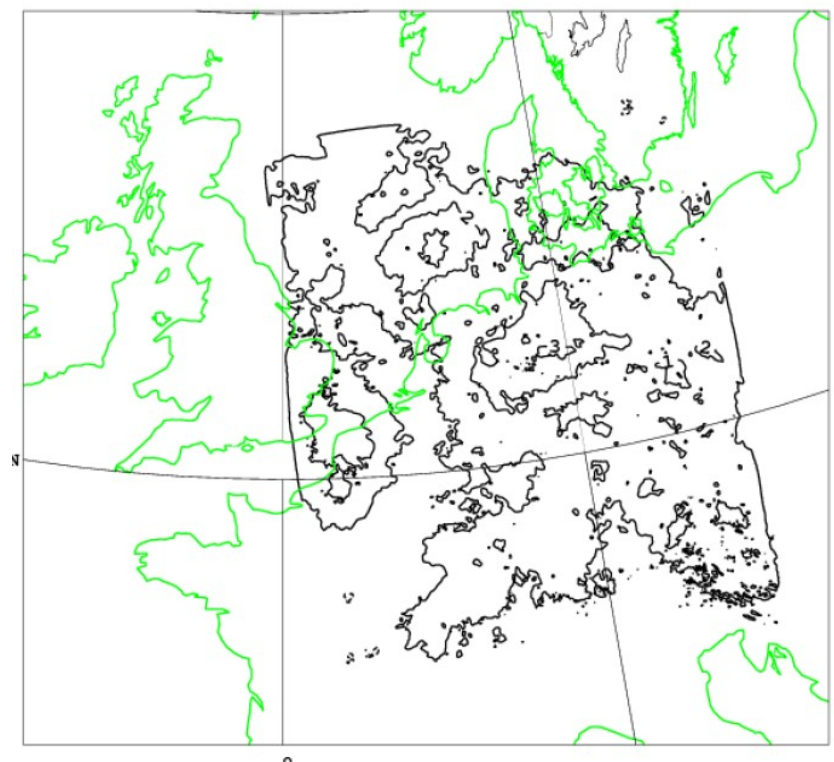


Evolution of two random perturbations with structure of B-matrix covariance

Forecast length: +01h

Surface pressure increment **13 08 2012 04UTC**

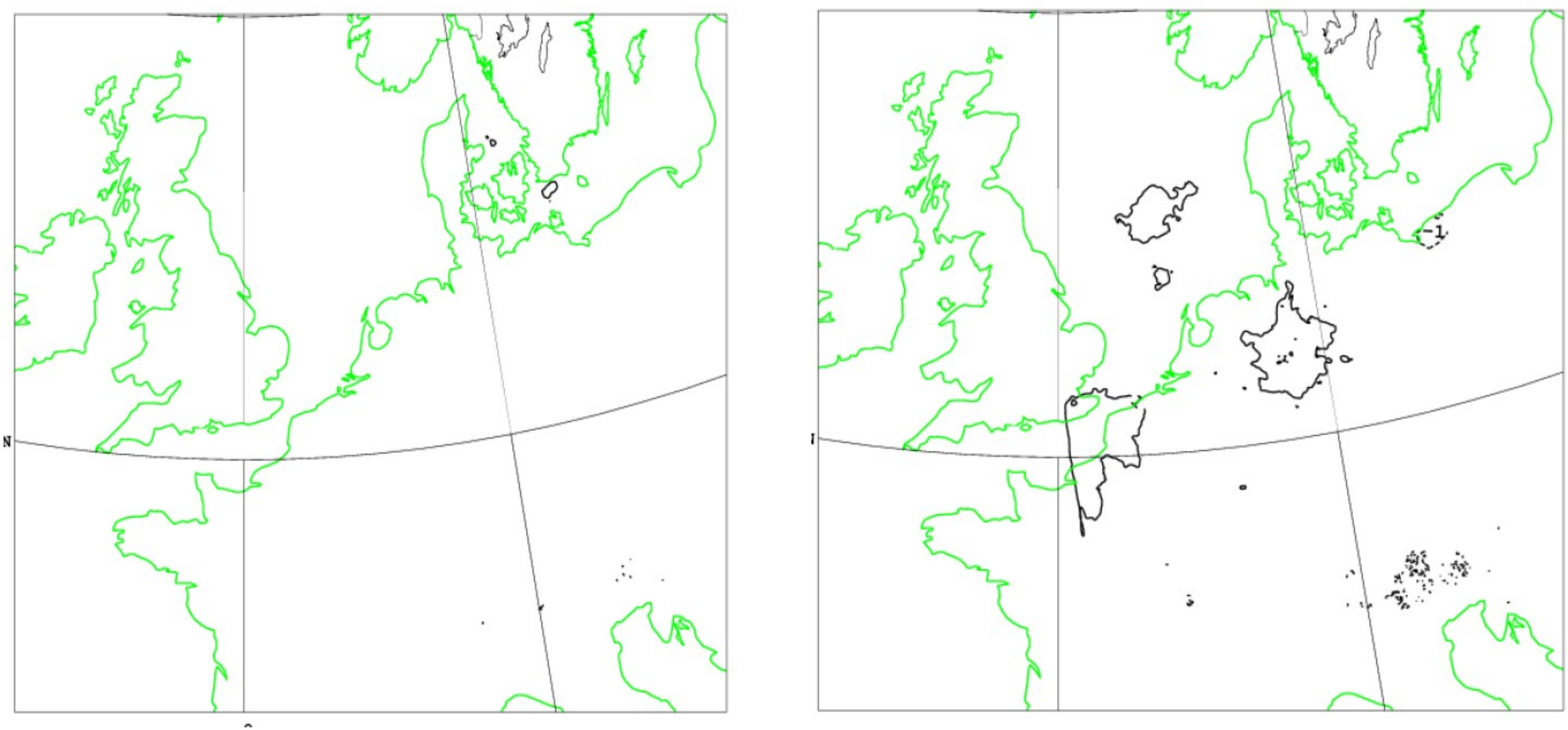




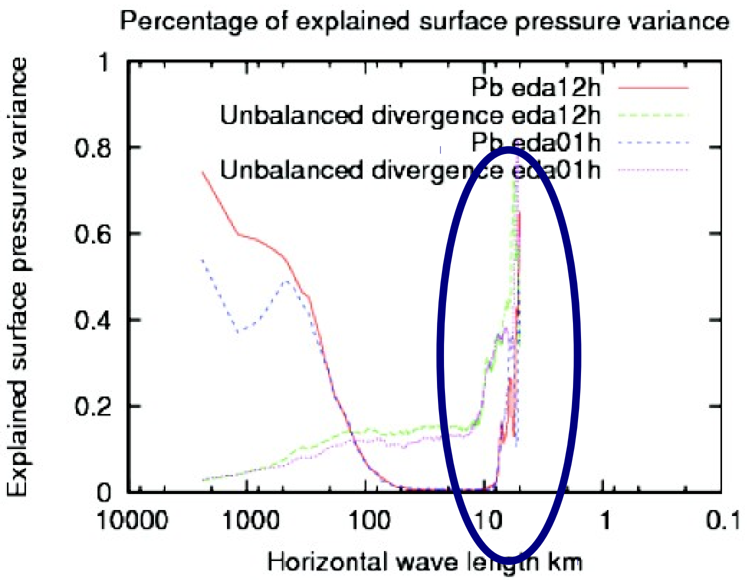
Evolution of two random perturbations with structure of B-matrix covariance

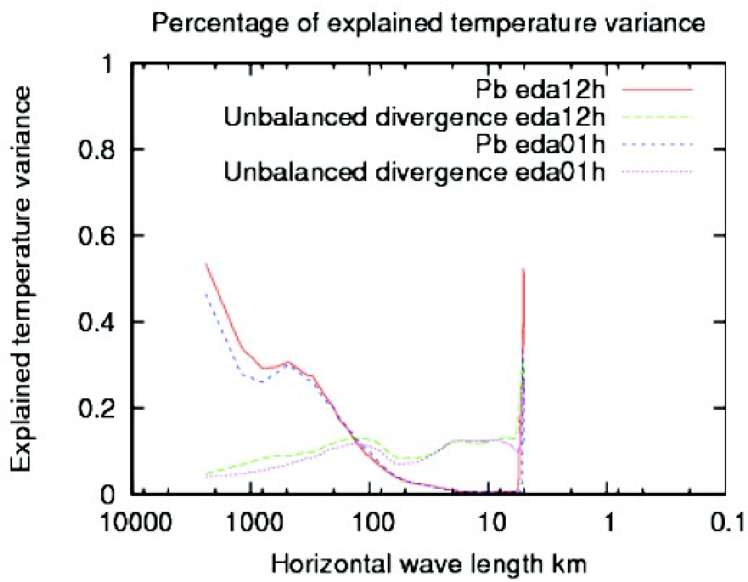
Forecast length: +05h

Surface pressure increment **13 08 2012 08UTC**



What structure functions say





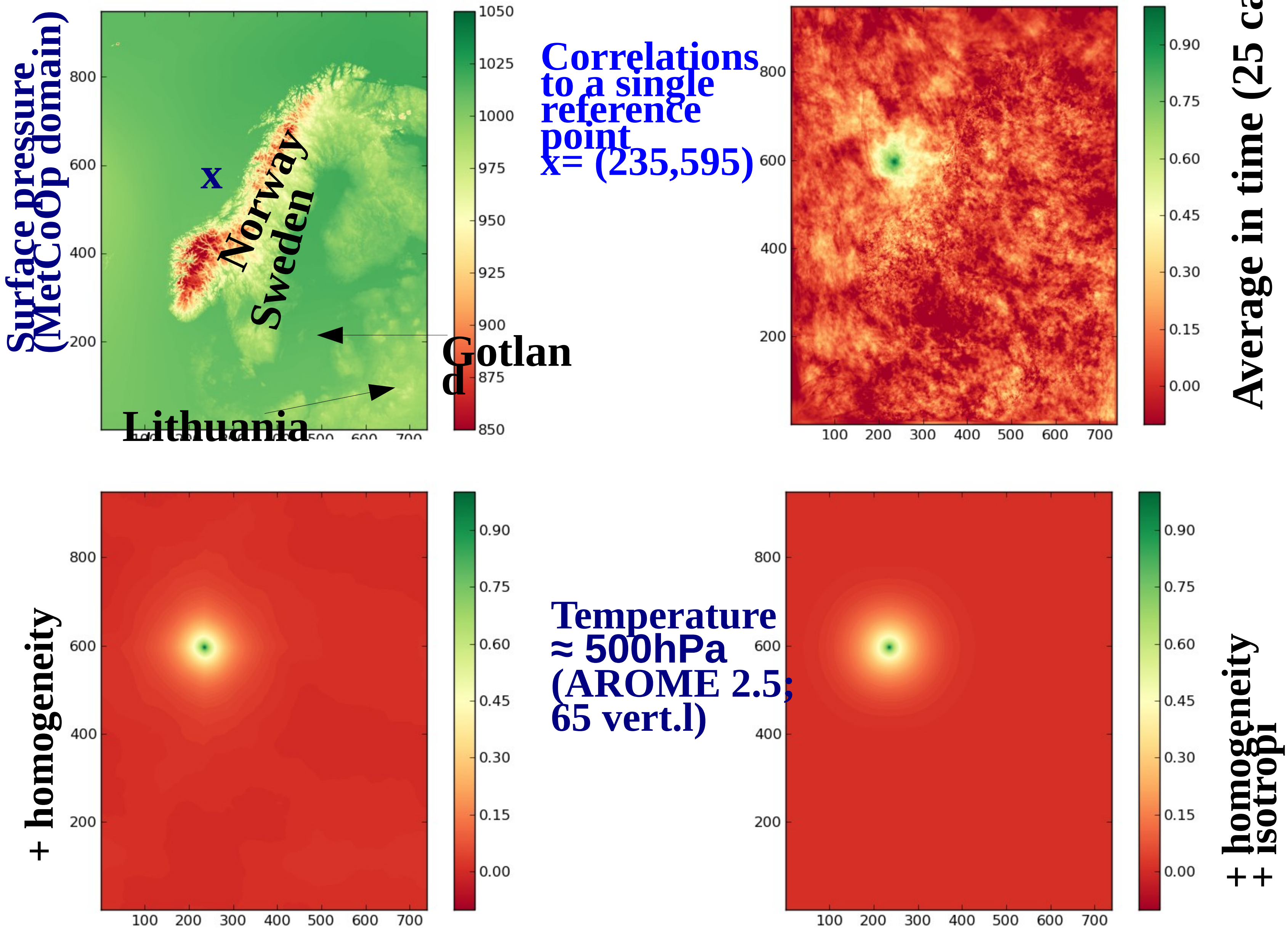
Aliasing of high-order terms on 2Δx, 3Δx, 4Δx, 5Δx waves

Obvious 2Δx problem

The preliminary results using **cubic grid truncation** (Mariano Hortal implementation) show encouraging results : **increased numerical stability of the scheme and longer time stepping in the semi-lagrangian forward propagation**. Processes are resolved in the grid-point space and smoothed in the spectral space.

Climatological structure functions

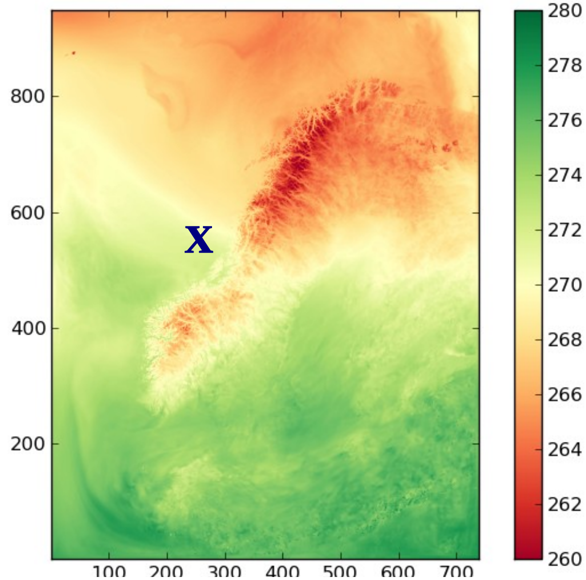
(6 EDA based HarmonEPS perturbations; 06UTC +12h)



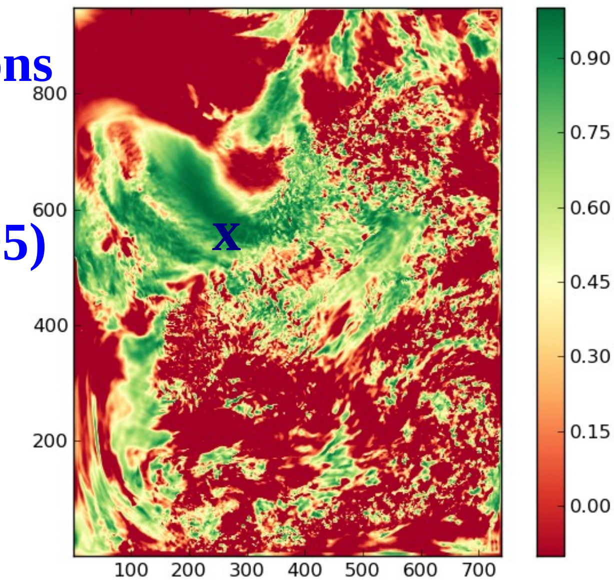
Climatological structure functions

(6 EDA based HarmonEPS perturbations; 06UTC + 12h)

Temperature (control)
(12 08 2008)

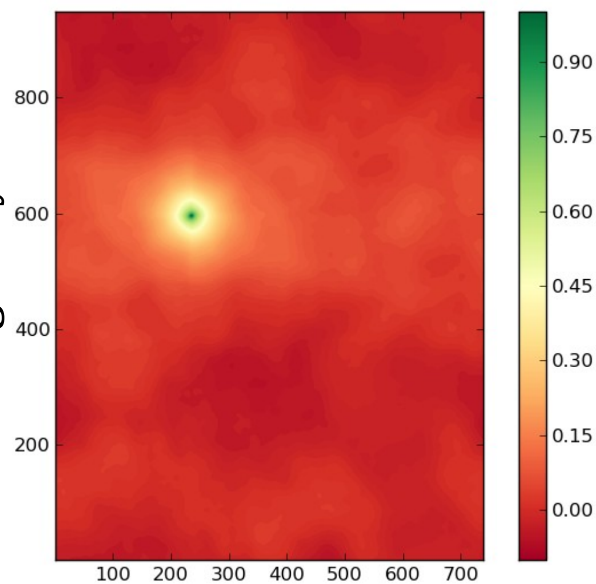


Correlations to a single reference point x= (235,595)

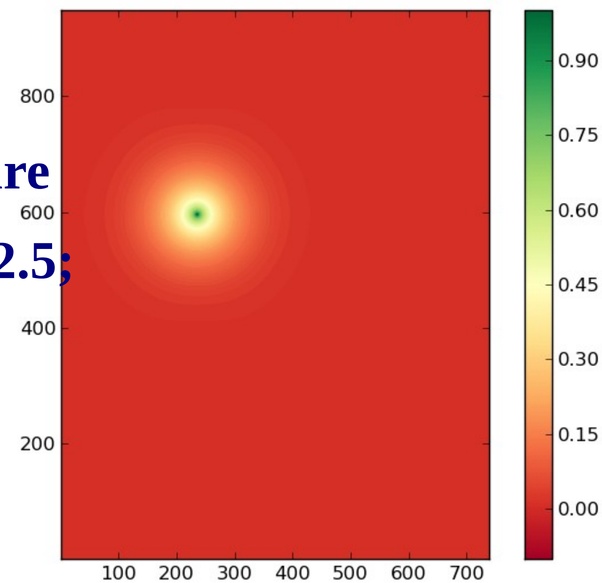


Single case

+ homogeneity



Temperature ≈ 500hPa (AROME 2.5; 65 vert.l)

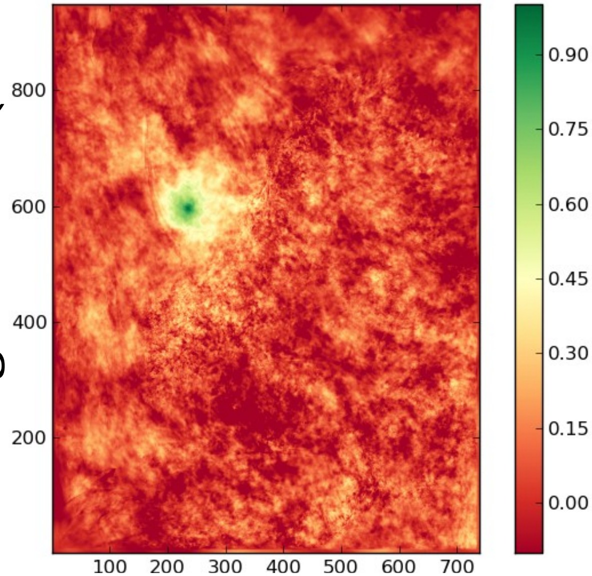


+ homogeneity
+ isotropy

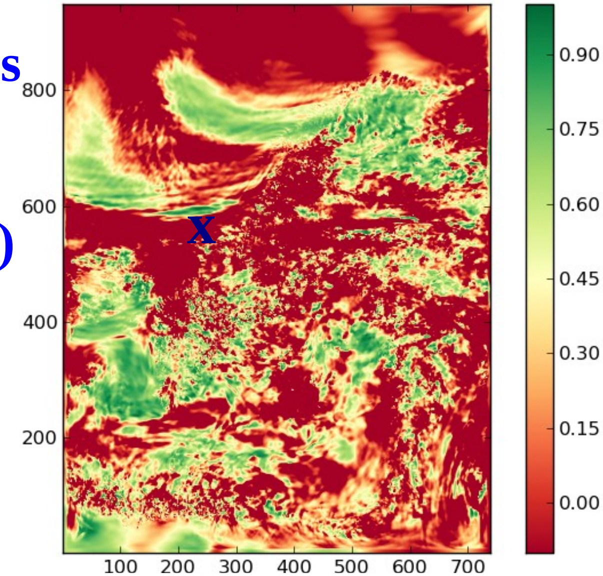
Climatological structure functions

(6 EDA based HarmonEPS perturbations; 06UTC + 12h)

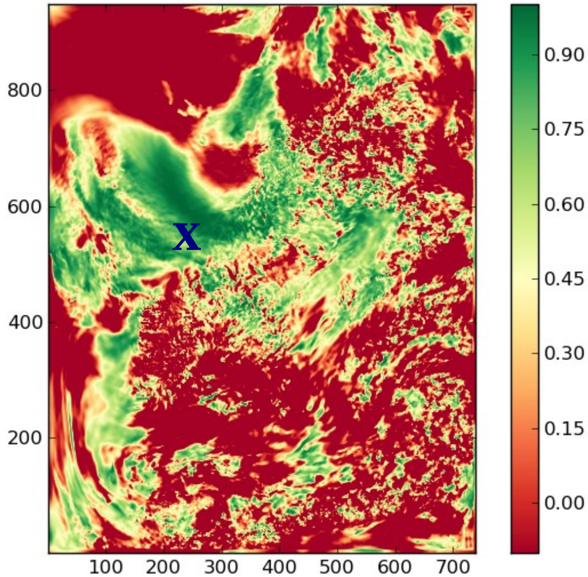
Average in time (25 cases)



Correlations to a single reference point x= (235,595)

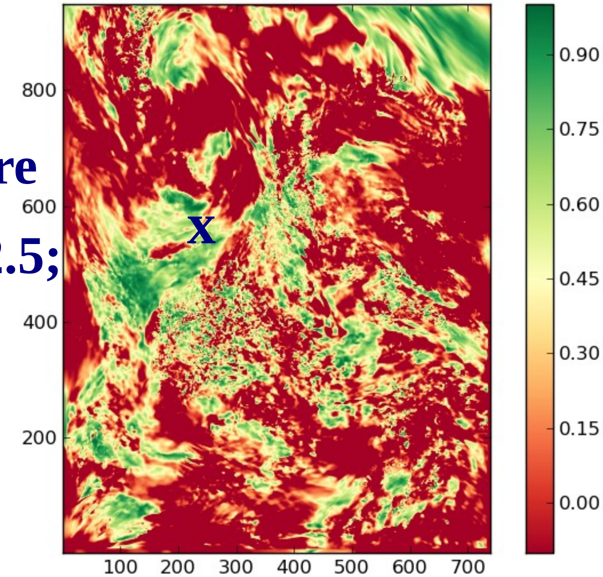


06 08 2008



12 08 2008

Temperature ≈ 500hPa (AROME 2.5; 65 vert.l)



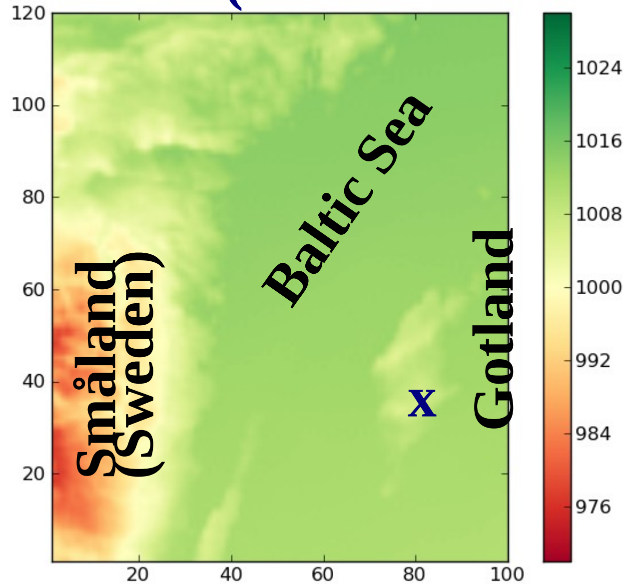
18 08 2008

Allow flow-dependency,
Avoid homogeneity & isotropi

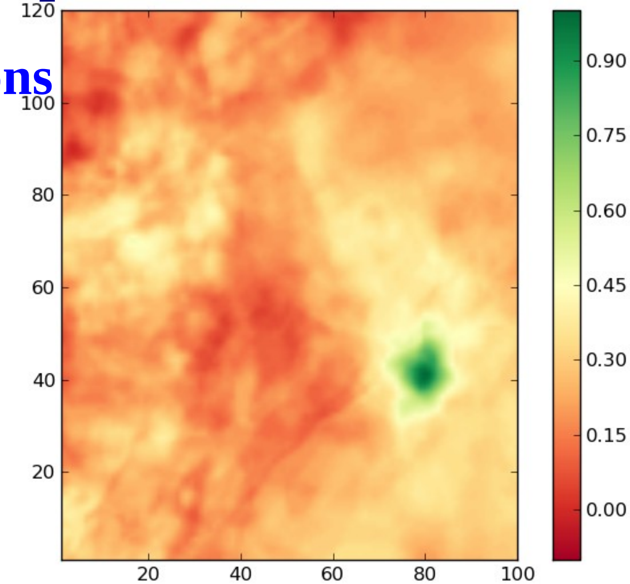
Climatological structure functions

(6 EDA based HarmonEPS perturbations; 06UTC + 12h)

Surface pressure (zoom over Gotland)

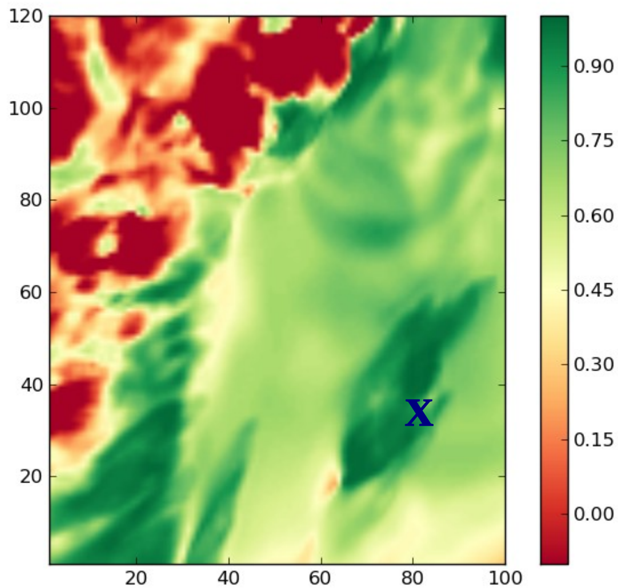


Correlations to a single reference point x= (80,40)

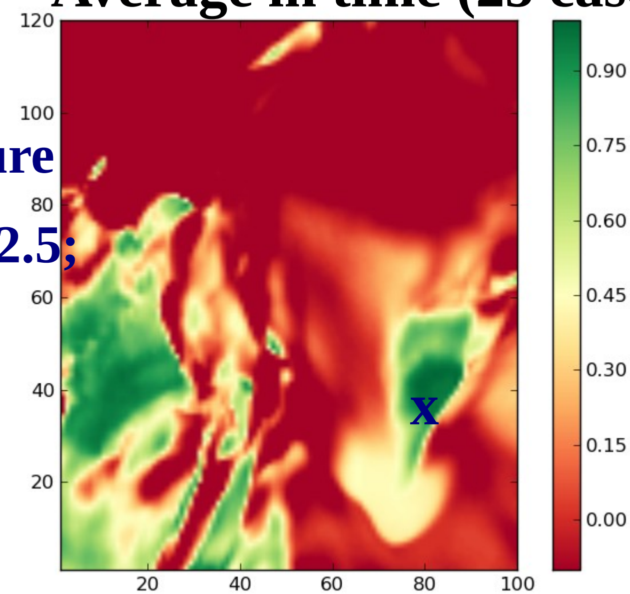


Average in time (25 cases)

Temperature level 62 (AROME 2.5; 65 vert.l)



12.08.2008



21.08.2008

Big potential of HarmonEPS ensemble for convective scale phenomena (surface &PBL)

What can we learn from this experiment:

1) **We cannot come much further forward without flow-dependent structure functions!=> homogeneity and isotropy assumption about the forecast error statistics do not hold for the convective scale phenomena;**

2) **Small scales structures and noise is a dangerous combination => Go for “cubic grid” truncation, possibly low-resolution orography; We need to rethink about initialisation on convective scales**

3) **Ensembles have big potential for data assimilation on convective scales (processes driven by surface and PBL conditions) => Go for Ensemble Variational techniques using convection permitting ensembles**

Status of NWP data assimilation developments

- ECMWF: Global 4D-Var. Add model error and ensemble components. EDA for EPS – variances and now also correlations. EnKF option.
- NCEP: Global and LAM 3D-Var. Flow-dependent B=>Hybrids. 4D-En-Var.
- UKMO: Global and LAM 4D-Var. Hybrid En-Var => 4D-En-Var.
- M-France: Global 4D-Var. LAM 3D-Var. Ensemble B. 4D-En-Var
- DWD: Global 3D-Var. LAM nudging. Considers flow-dependent B globally and LAM EnKF.
- Japan: Global and LAM 4D-Var. Considers EnKF for the global model.
- Canada: 4D-Var for deterministic models, EnKF for EPS. Hybrids.4D-En-Var
- HIRLAM: 4D-Var. Hybrid En-Var. 4D-En-Var.
- ALADIN/HARMONIE: 3D-Var and development of 4D-Var. Ensemble components.
- University world: strong dominance for EnKF

3D-En-Var in an operational ocean model

(Lars Axell, SMHI, personal communication)

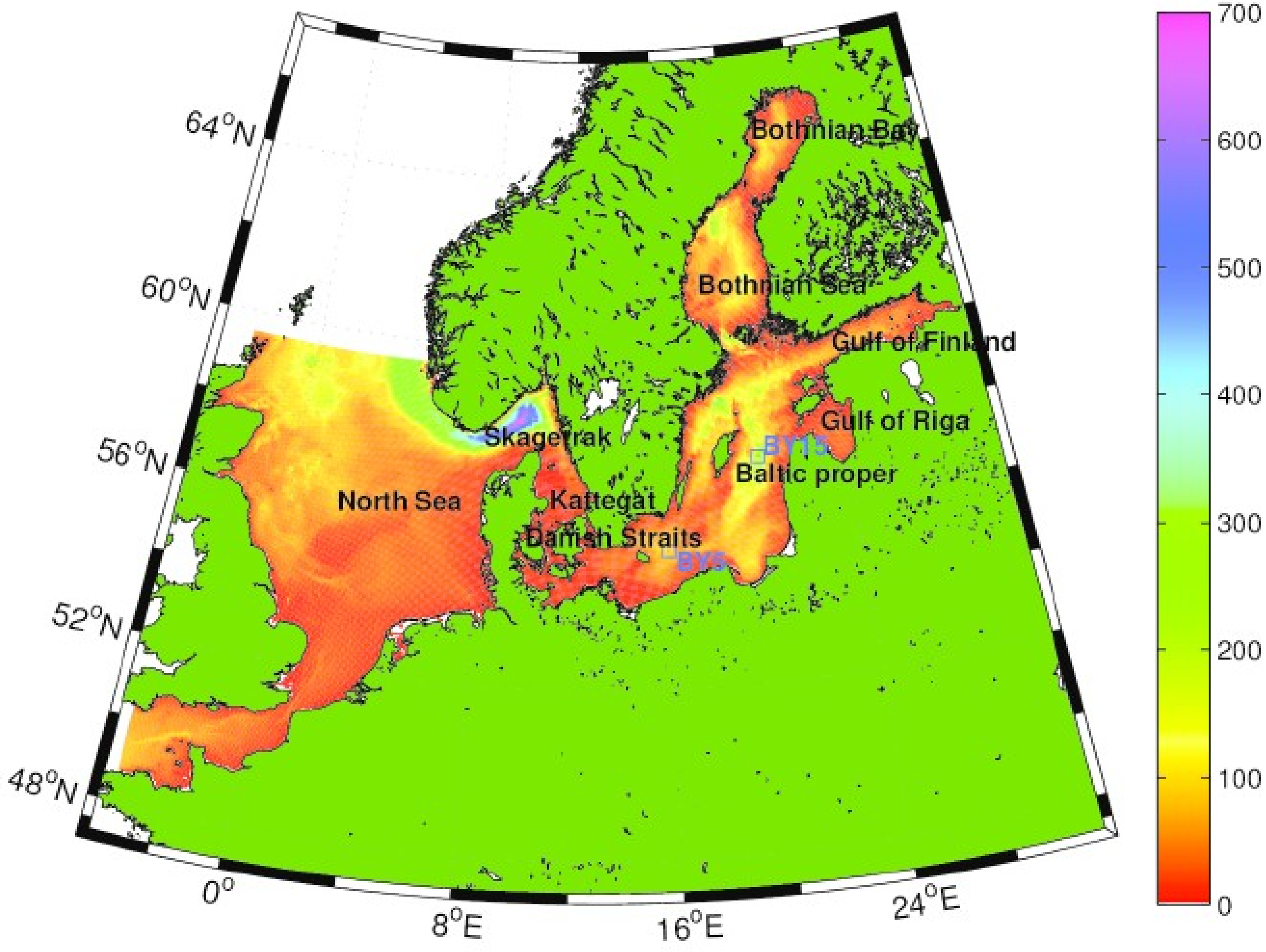
Some basic features

- HIROMB or NEMO models
- Minimization of a cost function
- Quasi-static ensemble (multi-year model integration; use of ensemble members from the same season)
- Localization function through EOFs and low resolution

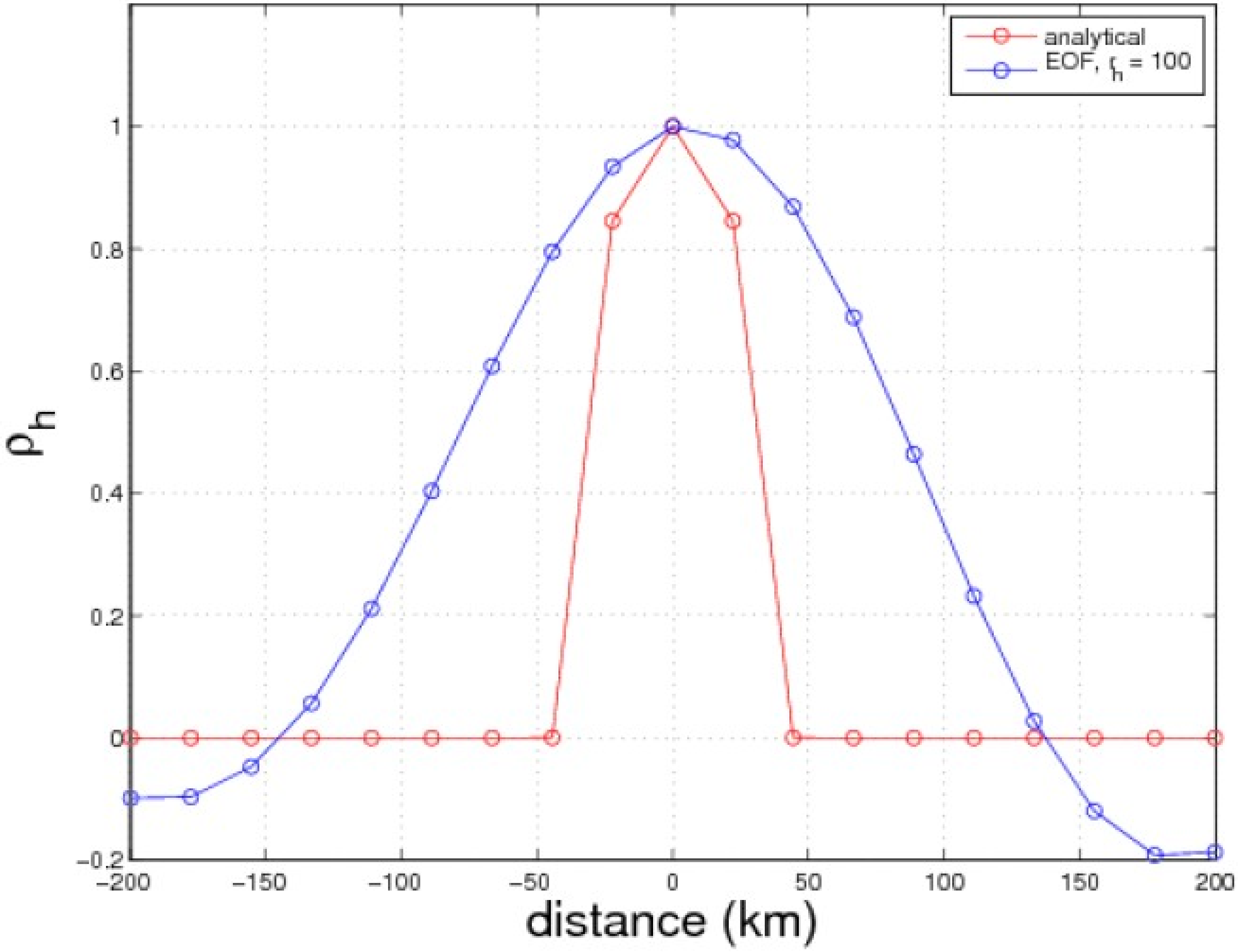
Why 3D-En-Var when an Ensemble OI is available?

- Non-linear observation operators
- Step toward (3)4D-En-Var with forecast ensemble

Operational HIROMB domain:

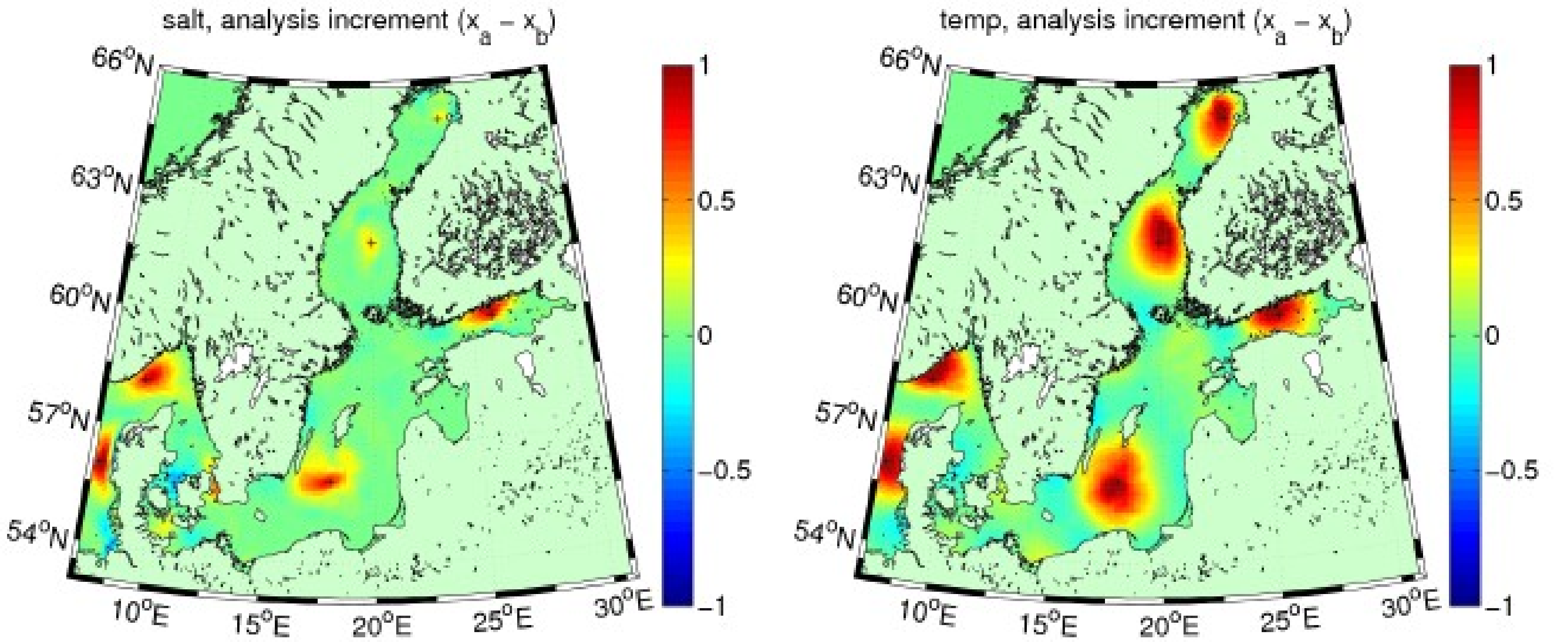


EOF truncation of localization function at low grid resolution



Single observation experiments

Surface salinity: Surface temperature:

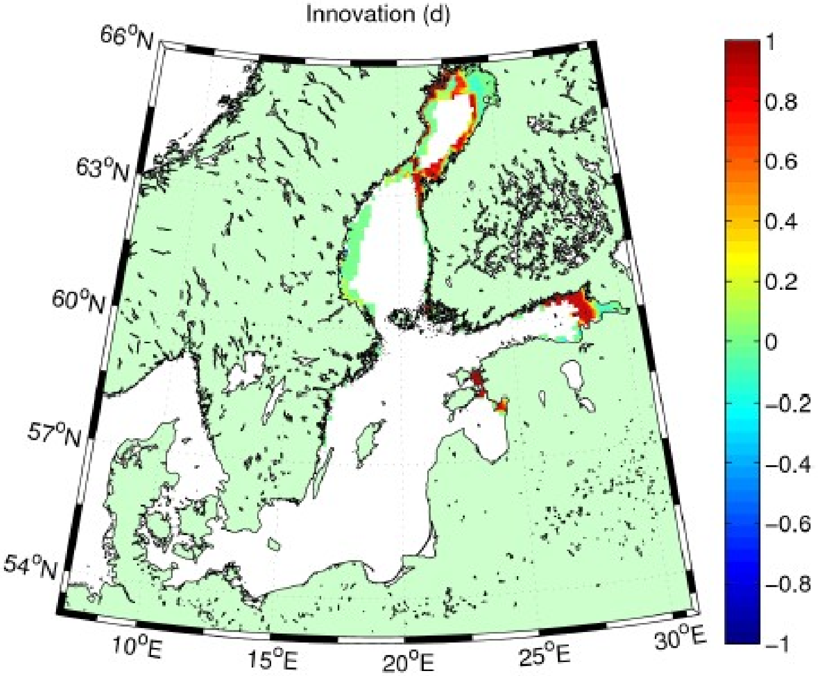


(a) (b)

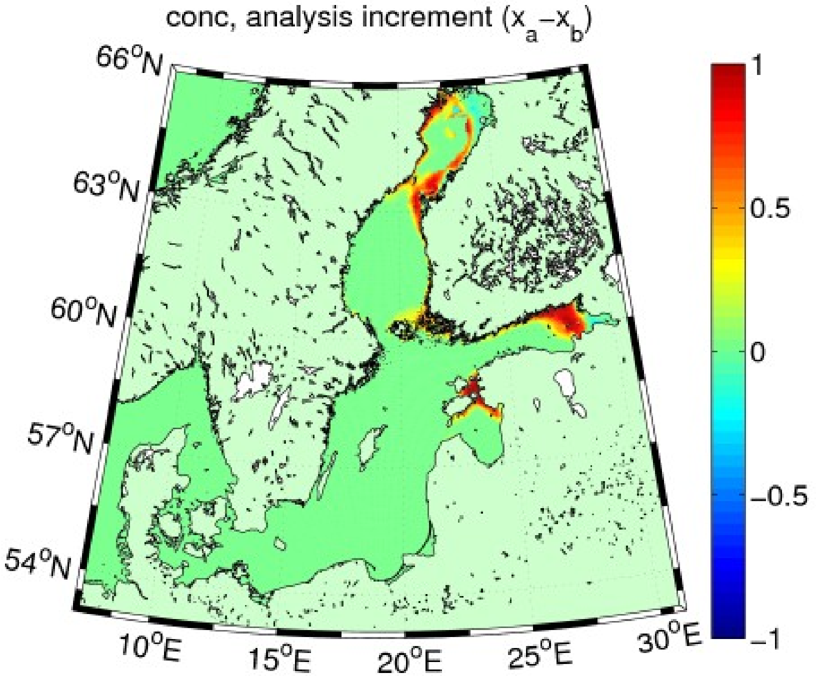
FIG. 4. Result from assimilating six synthetic observations of (a) SSS and (b) SST. The observation locations are indicated with crosses (+).

Assimilation of real observations, Sea Ice Concentration

Innovations:



Analysis increments:



25 years reanalysis;

Salinity profiles;

Dependent observations

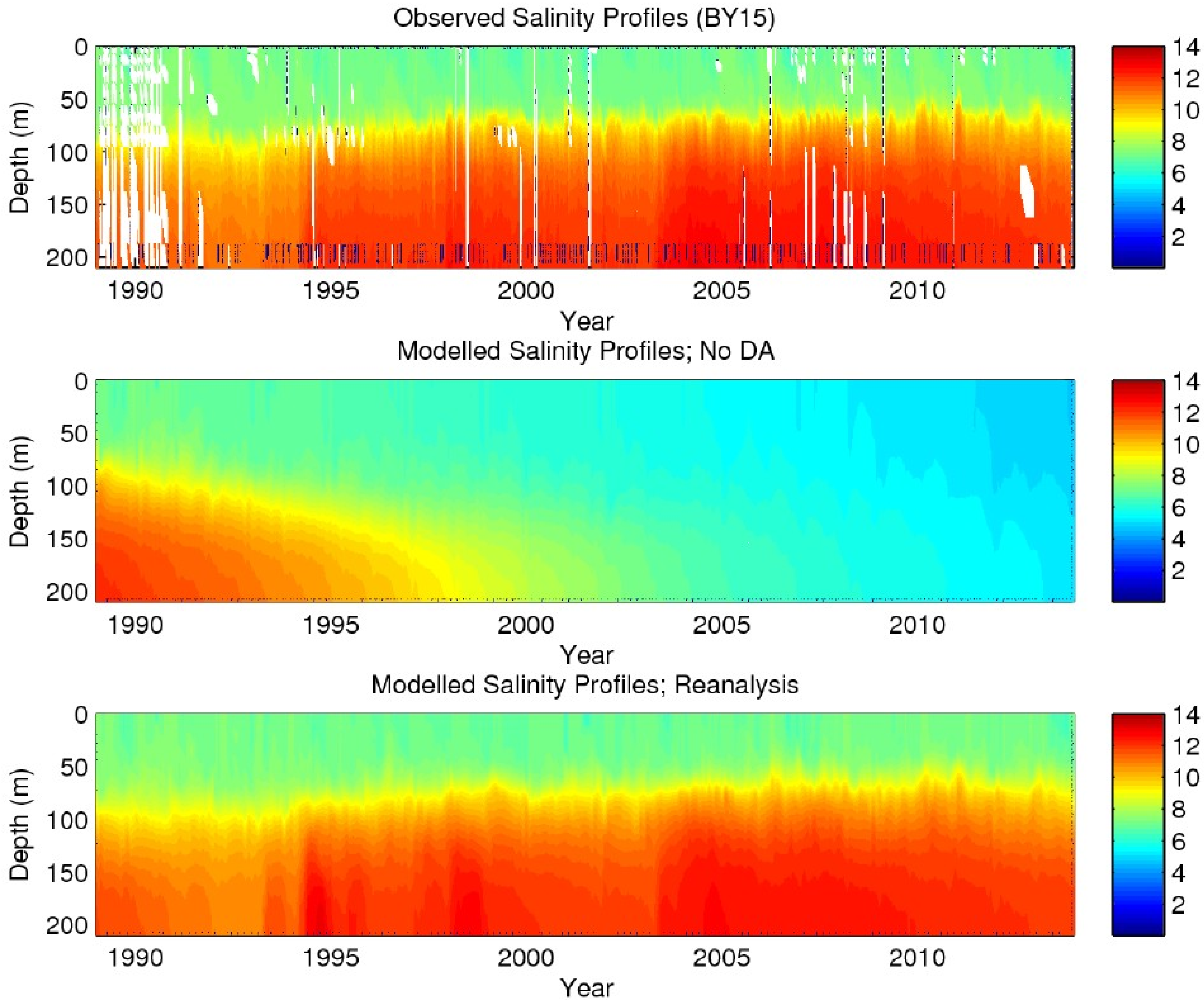


FIG. 12. Salinity profiles at station BY15, according to (a) observations, (b) a free run (without data assimilation), and (c) the reanalysis (with data assimilation). The data are sampled once a month.

25 years reanalysis;

verification against independent observations

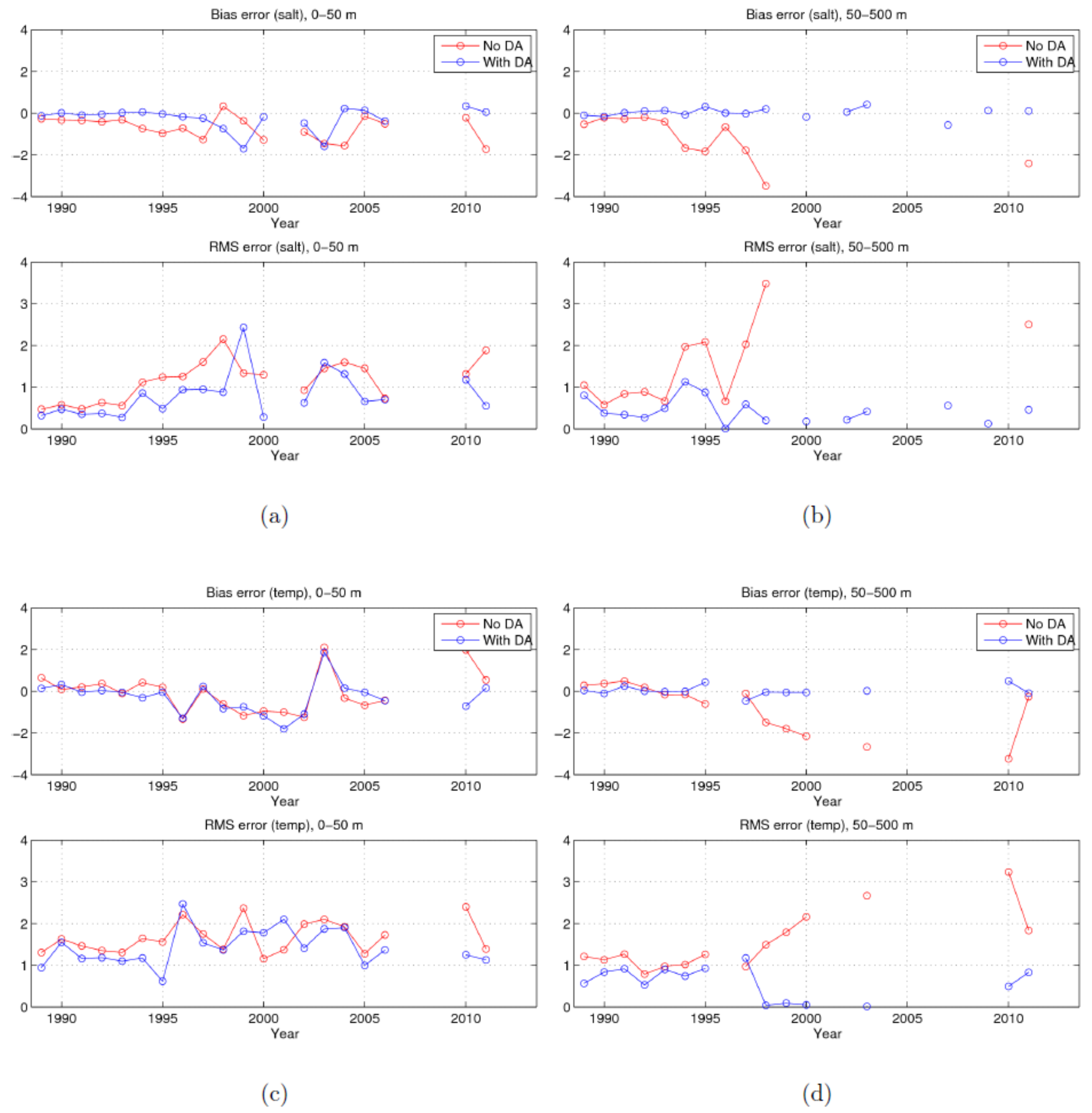


Fig. 14. Bias and rms errors from two 25-year simulations, one free simulation ("No DA") and a reanalysis run ("With DA"). The panels show (a) near-surface salinity (0-50 m), (b) sub-surface salinity (50-500 m), (c) near-surface temperature (0-50 m), and (d) sub-surface temperature (50-500 m).

Concluding remarks

Adding ensemble information provides improvements to 3D-Var and 4D-Var (Hybrid Var Ens) (ds=10km)

Replacing the TL /AD models in 4D-Var with a 4D ensemble (4D-En-Var) provides results comparable to 4D-Var Hybrid

Several factors contribute to difficulties for 3D-Var assimilation at meso scale (ds=2.5km)

- assumptions on stationarity, homogeneity and isotropy are not valid
- no initialization
- ensemble information has potential to help, localization becomes an important issue